2011 Numeracy Conference for Administrators

Schema Based Instruction

John Woodward
Professor, University of Puget Sound
Tacoma, Washington
What is Schema?

A tool for future learning

Schema eases the cognitive effort required

Textual/symbolic representations

Visual representations

Connections leading to other connections
Framework for Thinking about Math

Schema as a Part of Information Processing Psychology
Information Processing Psychology

- Declarative Knowledge
- Procedural Knowledge
- Strategic Knowledge
- Schematic Knowledge
Declarative Knowledge

• **Knowing What**
  – a fact-base, or knowledge store

• $4 \times 5 = 20$
• A quadrilateral has 4 sides
• The denominator in a fraction tells how many total parts there are in the whole
Procedural Knowledge

• “Knowing how to do something”
• Consists of…
  – if/then rules
  – sequence of steps
• Calls upon…
  – declarative knowledge
Procedural Knowledge

• **Knowing how**
  – I know how to use reciprocals in balancing an equation
  – I know how to find the surface area of a cylinder
  – I know how to make a proportion using an equivalent fraction algorithm
Traditional math instruction excels at:
  – Declarative and procedural knowledge

(But criticisms aside, it is still important)
The Limits of Procedural Instruction

In this empirical study, the Procedural Group was:

• Given instruction with high fidelity of implementation

• Held to an average daily percent correct on workbook material of 78% over 4 weeks

• Assessed weekly as the completed the course

• Given a delay in post testing of 10 days

Procedural Knowledge After 10 Days

• On a test of hand computation (mean = 34%)

• Compute: .27 x .15
  – 22% did this correctly

• Convert 5/8 to a decimal
  – 33% multiplied the numerator and denominator
  – 22% divided the numerator into the denominator

• Order: .089  3.245  .47  .6  11.2
  – 39% confused whole numbers and decimals

• Write as a decimal
  – 50% wrote 2.3 or .23
Information Processing Psychology

• Traditional math instruction excels at:
  – Declarative and procedural knowledge

• We don’t do so well at:
  – Strategic knowledge
  – Schematic knowledge
Strategic Knowledge

• Knowing when, where and how to use certain types of knowledge in a new situation
  – I can work backwards to solve a problem
  – I can divide a regular hexagon into equilateral triangles to find the area of the hexagon
Schematic Knowledge

- Having a justifiable conception, i.e. “knowing why”
- May be used to...
  - interpret problems
  - troubleshoot
  - explain an outcome
  - predict an outcome
- Depends on...
  - having an understanding of principles
- “conceptual knowledge” = combination of declarative and procedural knowledge
Schematic Knowledge

• I can explain how the formula for the area of a triangle is derived from a rectangle

• I know different underlying structures for addition and subtraction problems such as compare, separate, and join
Information Processing Psychology

Monitoring or Metacognition
A Web of Understanding (and Connections)


Supported by:

1. Screen Students
2. Interventions Stress
   - Whole Numbers Grades K-5
   - Rational Numbers Grades 6-8
3. Explicit and Systematic Instruction
4. Solve Word Problems Based on Common Structures
5. Visual Representations*
6. Teach Facts*
7. Monitor Progress
8. Include Motivational Strategies


From Facts to Fractions

- Operations on Whole Numbers
- Emphasis on Place Value
- Number Decomposition
- Facts, Extended Facts, Powers of Ten
- Assimilate Operations on Fractions through Metacognition
- Textual/Symbolic representations
- Visual representations
6 Teach Facts

Traditional Method for Teaching Number Combinations or “Facts”

- Facts are learned through extensive practice
- Facts exist to support the traditional algorithms
Why is this so difficult for so many (American) students?
Just Beyond the Facts

90
x  6

Why would so many (American) students need paper and pencil to solve this problem?
And We Want Students to Know Facts Because …..

\[
\begin{array}{c}
594 \\
\times 36 \\
\hline
3564 \\
+ 1782 \\
\hline
21384
\end{array}
\]

Don’t calculators and computers do this?
Here’s a Common Structure for Teaching Facts

12  12  12  12  12  12  12  12
-  3    -  4    -  5    -  6    -  7    -  8    -  9

13  13  13  13  13  13  13
-  4    -  5    -  6    -  7    -  8    -  9

14  14  14  14  14  14
-  5    -  6    -  7    -  8    -  9
How to Think about This Kind of Instruction

It Places Large Demands on **Associative Memory**
Here Are Common Responses by Low Ability Students to This Kind of Structure

\[
\begin{align*}
12 & - 7 \\
\_ & \_ \\
6 & \\
\text{(or 4)} & \\
\end{align*}
\]
These Reasons May Explain Failure

• Teaching facts through assessment only

• Teaching facts as isolated bits of knowledge (the associative memory problem)

• Teaching facts as disconnected from the development of number sense
Teaching Facts through Assessment Only

- The number of new facts may overwhelm a student
- There’s little built into “know these facts by next _____” that maintains review
- There’s almost no opportunity to see strategies, to see how facts connect with other number sense skills
### Facts and Associative Memory

<table>
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<th>9 x 6 =</th>
<th>14</th>
<th>35</th>
<th>54</th>
<th>72</th>
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<tbody>
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<td>6 x 7 =</td>
<td>18</td>
<td>36</td>
<td>56</td>
<td>77</td>
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<td>7 x 7 =</td>
<td>21</td>
<td>42</td>
<td>63</td>
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<td>8 x 7 =</td>
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<td>9 x 5 =</td>
<td>28</td>
<td>49</td>
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<td>90</td>
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<td>8 x 8 =</td>
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Facts and Strategies and Number Sense

| 1 x 9 = 9 |
| 2 x 9 = 18 |
| 3 x 9 = 27 |
| 4 x 9 = 36 |
| 5 x 9 = 45 |
| 6 x 9 = 54 |
| 7 x 9 = 63 |
| 8 x 9 = 72 |
| 9 x 9 = 81 |

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| 4 x 9 = 36 |
| 5 x 9 = 45 |
| 6 x 9 = 54 |
| 7 x 9 = 63 |
| 8 x 9 = 72 |
| 9 x 9 = 81 |
Assumptions About Structure

Students Need

• Strategies
• Visual Representations
• Controlled Practice Opportunities
  – The amount of new information
  – Distributed practice
• Connections
Strategies Plus Visual Representations

The Ten Frame
Addition Facts

2 + 2
Addition Facts

2 + 2

“doubles”
Addition Facts

3 + 3

“doubles”
Addition Facts

3 + 3

“3 + (2 + 1)”
## Addition Facts

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</tbody>
</table>

4 + 5

“doubles + 1”
Addition Facts

8 + 9

“doubles + 1”
Addition Facts

8 + 9

“through 10s or 8 + 2 + 7”
Subtraction Facts

8 - 4

“doubles”
Subtraction Facts

9 – 7 or 9 - 2  “difference of 2”
Subtraction Facts

12 - 7

“through 10s or 12 - 2 - 5”
12 - 7

“through 10s or 12 - 2 - 5”
Subtraction Facts

The Power of the Through 10s Strategy

\[
\begin{array}{cccccccc}
12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 \\
- 3 & - 4 & - 5 & - 6 & - 7 & - 8 & - 9 \\
13 & 13 & 13 & 13 & 13 & 13 & 13 \\
- 4 & - 5 & - 6 & - 7 & - 8 & - 9 \\
14 & 14 & 14 & 14 & 14 & 14 \\
- 5 & - 6 & - 7 & - 8 & - 9 \\
\end{array}
\]
How to Think about This Kind of Instruction

Strategies Enhance Semantic Memory
Multiplication Facts

6 x 7 as 6 x 6 + 6 more

0 6 12 18 24 30 36 42

7 x 8

0 7 14 21 28 35 42 49 56
Strategies

• They run through almost all of the facts
  – There are a few outlaws that need to be isolated

• They allow for active engagement

• They can only be presented, not forced
Addition

• Rules: + 0, + 1, + 2
• Commutation

• Doubles (3 + 5 is the same as 4 + 4)
• Doubles + 1 (4 + 5 is 4 + 4 + 1)
• Through 10 (8 + 5 is 8 + 2 + 3)
Subtraction

- Rules: -0, -1, -2
- Find the difference or “add up”
  (9 – 7 we count up from 7)
- Through 10
Multiplication

- Rules: $x \ 1, \ x \ 2$
- Doubles (7 $x$ 2, 5 $x$ 2)
- Times Themselves (3 $x$ 3, 4 $x$ 4)
- The Nifty Nines
- Helping Facts (7 $x$ 6 is 6 $x$ 6 $+$ 6)
Division

- Multiplication and Division are Families
  32, 8, and 4 are related

- Near facts
  16 ÷ 4 and 17 ÷ 4, 18 ÷ 4, 19 ÷ 4
Different Visual Representations

- Ten frames
- Fact family triangles
- Arrays
- Number lines
Ten Frames

$8 + 5 = 8 + 2 + 3$ (thru 10s)
Arrays

$3 \times 4$
Division and the Number Line

24 ÷ 6
Timed Practice Set

4   3   2   7   1
+ 5 + 4 + 3 + 1 + 8
9   8   2   4   1
+ 1 + 8 + 3 + 4 + 2
1   1   5   1   0
+ 5 + 2 + 4 + 6 + 1

2 pages of 20 facts per page – 2 minute time limit – 36 correct
Structure of Timed Practice Drills

- Present Strategies in Small Sets
  "7 +/- 2"
- Provide Distributed Practice
- Provide Timed Practice
  50% new set, 50% review

The latter insures high levels of success
Connecting Facts to Extended Facts

\[
\begin{array}{cccccc}
1 & 10 & 2 & 200 & 3 \\
+2 & +20 & +1 & +100 & +4 \\
40 & 2 & 30 & 1 & 10 \\
+30 & +3 & +20 & +0 & +0 \\
4 & 400 & 2 & 10 & 4 \\
+5 & +500 & +1 & +20 & +3 \\
30 & 3 & 200 & 5 & 40 \\
+40 & +2 & +300 & +4 & +50 \\
\end{array}
\]
# Rounding and Approximation

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<th>+30</th>
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<td>55</td>
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We’re on the freeway. I set cruise control for 62 miles per hour. About how far do we drive after 3 hours?

\[ 60 \times 3 \]  

Extended Facts
Building Connections to Operations

- Alternative Algorithms Showing Place Value
- Visual Representations Showing Underlying Concepts
Conceptually Guided Algorithms

Addition

\[
\begin{array}{c}
366 \\
+ 247 \\
\hline
613
\end{array}
\]

Subtraction

\[
\begin{array}{c}
366 \\
- 247 \\
\hline
119
\end{array}
\]

\[
\begin{array}{c}
300 \\
+ 200 \\
\hline
500
\end{array}
\]

\[
\begin{array}{c}
300 \\
- 200 \\
\hline
100
\end{array}
\]
Horizontal Addition and Place Value

366 + 247

366 + 247

\[300 + 60 + 6 + 200 + 40 + 7\]

\[300 + 200 + 60 + 40 + 6 + 7\]

\[500 + 100 + 13\]

613
Multiplication and the Emphasis on Place Value: The Partial Product Algorithm

\[
\begin{array}{c}
357 \\
\times 4 \\
28 \\
200 \\
+ 1200 \\
\hline
1428
\end{array}
\]

or

\[
\begin{array}{c}
357 \\
\times 4 \\
1200 \\
200 \\
+ 28 \\
\hline
1428
\end{array}
\]
How *would you* explain the problem conceptually to students?
Accommodating New Structures

Operations on Whole Numbers
Emphasis on Place Value

Assimilate Operations on Fractions through Metacognition

Number Decomposition
Facts, Extended Facts, Powers of Ten

Textual/ Symbolic representations
Visual representations
Addition and Subtraction

Operations and Guiding Questions

3 + 4 = 7

As long as we count by 1, this seems fairly easy. No questions asked.
So why can’t you add $\frac{1}{4}$ and $\frac{1}{3}$ the same way?
Addition and Subtraction of Fractions

Operations on Fraction and Fraction bars

\[
\frac{1}{4} + \frac{2}{4} = \frac{3}{4}
\]
Addition and Subtraction of Fractions

Fraction bars and subtraction

\[ \frac{3}{4} - \frac{1}{4} = \frac{2}{4} \]
Multiplication

When you multiply two numbers, the product is usually larger than either of the two numbers.

3 x 4 = 12
When you multiply two fractions, the product is usually smaller. Why?

\[ \frac{1}{4} \times \frac{2}{3} = \frac{2}{12} \text{ or } \frac{1}{6} \]
\[ \frac{1}{4} \times \frac{2}{3} = \]
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$\frac{1}{4} \times \frac{2}{3} = \frac{2}{3}$
$1/4 \times 2/3 = \frac{2}{3}$
$\frac{1}{4} \times \frac{2}{3} = $ 

Answer = $\frac{2}{12}$
Division

When you divide two numbers, the quotient is usually smaller than the dividend.

\[12 \div 4 = 3\]
Assuming We Stop to Notice

When you divide two fractions, the quotient is usually larger than the dividend. Why?

\[ \frac{2}{3} \div \frac{1}{2} = \frac{4}{3} \]
Division of Fractions

\[ \frac{8}{2} \rightarrow 2 \sqrt{8} \]

Doesn’t the unit of 2 partition 8 four times?
Division of Fractions

\[ 2 \div \frac{1}{2} \rightarrow \frac{1}{2} \sqrt{2} \]

Doesn’t the unit of \(\frac{1}{2}\) partition 2 four times?
Division of Fractions

\[ \frac{3}{4} \div \frac{1}{2} \]

Doesn’t the \( \frac{1}{2} \) partition the \( \frac{3}{4} \) \( 1 \) and \( \frac{1}{2} \) times?
Division of Fractions

\[ \frac{3}{4} \div \frac{1}{2} \]

1 time

1 time

\[ \frac{1}{2} \]

\[ \frac{3}{4} \]
Division of Fractions

\[
\frac{3}{4} \div \frac{1}{2}
\]

1 time and \( \frac{1}{2} \) time

\[
\frac{1}{2} \quad \frac{3}{4}
\]
Eva spent \( \frac{2}{5} \) of the money she had on a coat, then spent \( \frac{1}{3} \) of what was left on a sweater. She had $150 remaining. How much did she start with?
Eva spent $\frac{2}{5}$ of the money she had on a coat, then spent $\frac{1}{3}$ of what was left on a sweater. She had $150 remaining. How much did she start with?
Eva spent $2/5$ of the money she had on a coat, then spent $1/3$ of what was left on a sweater. She had $150$ remaining. How much did she start with?

She spent $2/5$ of her money on a coat

She had $3/5$ remaining after buying the coat
Eva spent 2/5 of the money she had on a coat, then spent 1/3 of what was left on a sweater. She had $150 remaining. How much did she start with?

She spent 2/5 of her money on a coat

She spent 1/5 of her money on a sweater

She had 2/5 remaining after buying the coat & the sweater. This portion is $150

½ of $150 = $75. That means 1/5 = $75

5 x $75 = $375