Preschool Flipbook

Early learning standards for Mathematics

This "flip book" is designed as a resource tool to assist teachers in deepening their understanding of what each standard means in terms of what students must know and be able to do. It provides only a sample of instructional strategies and examples. Every teacher must guide students in understanding and making sense of the mathematics that are presented. Young children need adult support and instruction, not only to develop surface level understanding, but also to deepen their early knowledge through the meaningful use of mathematical concepts in everyday situations. Links to conceptual categories and specific standards in the document can be accessed from page ?

The Preschool flipbook was adapted by Carla Heintz, USD #259 and Chelie Nelson, KITS, from the Kindergarten flipbook created by Melisa Hancock (melisa@ksu.edu).

Resources Used:

Clements, D. H., & Sarama, J. (2009). *Learning and Teaching Early Math: The Learning Trajectories Approach*. New York, New York: Routledge.

Common Core State Standards for Mathematics
http://www.ksde.org/LinkClick.aspx?fileticket=NrF2sZEzKQI%3d&tabid=5276&mid=13067

Fuson, K. C., Clements, D. H., & Beckman, S. (2010). *Focus in Prekindergarten: Teaching with Curriculum Focal Points*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Kansas Early Learning Standards weblink

Construction directions:
Print on cardstock. Cut the tabs on each page starting with page 2. Cut the bottom off of this top cover to reveal the tabs for the subsequent pages. Staple or bind the top of all pages to complete your flipbook.

Compiled by Carla Heintz and Chelie Nelson (Send feedback to chelie@ku.edu)

Mathematical Practice Standards (**MP**) adapted from Common Core State Standards

The Standards for Mathematical Practice from the Common Core State Standards describe varieties of expertise that mathematics educators at all levels (including early childhood) should seek to develop in their students. These practices rest on important "process and proficiencies" with longstanding importance in mathematics education. The Standards for Mathematical Practice describe ways in which students ought to increasingly engage with the subject matter as they grow in mathematical maturity.

1. **Make sense of problems and persevere in solving them**.
   Mathematically proficient students interpret and make meaning of problems looking for starting points. In preschool, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
2. **Reason abstractly and quantitatively**.
   Mathematically proficient students make sense of quantities and their relationships. Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
3. **Construct viable arguments and critique the reasoning of others**.
   Mathematically proficient students analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. They justify conclusions with mathematical ideas. They listen to the arguments of others and ask useful questions to determine if an argument makes sense or suggest ideas to improve/revise the argument. Students begin to construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. **Model with mathematics**.
   Mathematically proficient students understand that models are a way to reason quantitatively and abstractly (able to decontextualize and contextualize). Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections.
5. **Use appropriate tools strategically.**
   Mathematically proficient students use available tools recognizing the strengths and limitations of each. Students begin to consider the available

tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. They use technological tools to deepen their understanding of mathematics.

6. **Attend to precision.**
   As students begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.
7. **Look for and make use of structure. (Deductive Reasoning)**
   Mathematically proficient students apply general mathematical rules to specific situations. They look for the overall structure and patterns in mathematics.
8. **Look for and express regularity in repeated reasoning. (Inductive Reasoning)**
   Mathematically proficient students see repeated calculations and look for generalizations and shortcuts. Students notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is one more. In addition, students continually check their work by asking themselves, "Does this make sense?"

Mathematical Practice Standards	How teachers can support the development of mathematical thinking
Makes sense of problems and persevere in solving them	Model a variety of ways to solve problems Provide materials for students to use when problem solving (e.g. such as manipulatives) or supplies for drawing or making a graph Ask questions to promote problem solving, such as: What are some other strategies you might try? What did you do to figure that problem out? What do you know about this problem? What are you trying to find out?
Reason abstractly and quantitatively	Model a variety of ways to represent problems (e.g. acting out, manipulatives, drawing, graphs). Discuss quantities (e.g. more, less, same and different). Make connections between written numerals and the quantities they represent Ask questions such as: How is ___ related to ____? What do you know about the numbers in the problem? How can we find the solution? What does ____ mean to you? (symbol, quantity, diagram)
Construct viable arguments and critique the reasoning of others.	Model ways to test that the solution you found works. Ask questions such as: How can we be sure that...? How could you prove that...? Will it still work if...? What were you thinking about when...? How did you decide to try that strategy? What is the same and what is different about...?
Model with mathematics	Model a variety of ways to represent problems (e.g. acting out, manipulatives, drawings, graphs) Ask questions such as: What can you do to visually represent this problem? Would it help to ....(draw a picture, graph, use manipulatives)?
Use appropriate tools strategically	Model a variety of tools to solve problems, and make comparisons including those for standard and non-standard measurement, manipulatives, and technology. Ask questions such as: What information do you have? What approach are you considering using? Is there something you could use that would help you find a solution? Why was it helpful to use…?
Attend to precision	Model mathematical vocabulary throughout the day. Provide opportunities throughout the day for students to participate in mathematical discussions. Ask questions such as: How did you come to that conclusion? How do you know your answer works to solve the problem? Tell me how you did that.
Look for and make use of structure	Point out patterns beyond simple linear patterns of colors. Look at visual patterns that occur in subitizing; the repetition of numbers

	that occur as you count in teens, twenties and beyond; that 3+2= 5 and 2+3 =5, and through ordinal numbers when describing linear patterns. (e.g. " every third block is blue") Ask questions such as: What did you notice when…? What patterns do you see? What observations can you make about…? How do you know if something is a pattern?
Look for and express regularity in repeated reasoning	Help students understand and use arithmetic generalizations, such as: when you add zero to a number the sum is always that number, and when you add 1 to a number the sum is always the next number in the counting sequence. Ask questions such as: What do you notice about…? What would happen if…? What predictions can we make based on this pattern?

Critical Areas for Mathematics in Preschool

In Preschool, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Preschool should be devoted to number than to other topics.

1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects. (Preschool students should see addition and subtraction equations written out, and be encouraged to solve them using objects.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Domain: **Counting and Cardinality**

Cluster: Know number names and the count sequence

Standards: **M.CC-NCS.4.1**
Counts in sequence to 30

Standards for Mathematical Practice (SMP) to be emphasized:
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Explanations and Examples:
M.CC-NCS.4.1 The emphasis of this standard is on the counting sequence. When counting by ones, students need to understand that the next number in the sequence is one more. Students are to rote count (verbal saying of numbers in sequence) by starting at one and counting to 30. This objective does not require recognition of numerals. It is focused on the rote number sequence.

Provide settings that connect mathematical language and symbols to the everyday lives of preschoolers. Support students' ability to make meaning and mathematize (to focus on the mathematical aspects of a situation and then to formulate that situation in mathematical terms to deepen, extend, elaborate and refine their thinking and reasoning) the real world. For example, students mathematize when they notice there are 3 birds rather than just "some" birds. Help them see patterns, make connections and provide repeated experiences that give students time and opportunities to develop understandings and increase fluency. Encourage students to explain their reasoning by asking probing questions such as —"How do you know?" —"How did you figure that out?"

Instruction on the counting sequence should be scaffolded (e.g. 1-5, then 5-10, 10-20, etc.)

Counting should be reinforced throughout the day, not in isolation. (Meaningful Counting)
Examples:
Count the number of chairs of the students who are absent
Count the number of stairs, shoes, etc.
Counting groups of items such as two eyes, two hands, two shoes.
Count the number of students in a group.
Count the number of items they have at snack

Accuracy in counting depends on three things:
1. Knowing the patterns in the number-word list so that a correct number-word list can be said
2. Correctly assigning one number word to one object (one-to one-correspondence)
3. Keeping track of which objects have already been counted so that they are not counted more than once. This is more easily done by moving objects into a counted set. This is not possible for things that cannot be moved such as pictures in a book,

however it is still important that all students practice counting pictures of objects as well as objects to generalize what they can count and extend their accurate counting to larger sets. Strategies for keeping track of messy, large sets continue to develop for many years.

Regularity and rhythm are important aspects of counting. Activities that increase these aspects can be helpful to students making lots of correspondence errors.

Errors in Counting
Four factors strongly affect accuracy in counting correspondence:
- Amount of counting experiences (more experience leads to fewer errors)
- Size of set (students become accurate on small sets first)
- Arrangements of objects (objects in rows make it easier to keep track of what has been counted and what has not)
- Effort

NCTM Focus in PreKindergarten, 2010

Common Misconceptions:
Some students might not see zero as a number. Ask students to say zero to represent the number of items left when all items have been taken away. Avoid using the word none to represent this situation. Find instances for which the response would be zero in real world settings to provide experiences with the concept of zero.

As long as students understand that correct counting requires one point and one word for each object and are trying to do that, parents and teachers do not need to correct errors all the time. But reminders to slow down can help them be more accurate. As with many physical activities, counting will improve with practice and does not need to be perfect each time. It is much more important for all students to get frequent counting practice, and to watch and help one another, with occasional help and corrections from the teacher.

Domain: **Counting and Cardinality (CC)**
Cluster: Know number names and the count sequence.
Standard: **M.CC-NCS.4.3** Count forward beginning from a given number (under 10) within the known sequence (instead of having to begin at 1).
Standards for Mathematical Practice (SMP) to be emphasized: MP.6 Attend to precision MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Explanations and Examples: The emphasis of this standard is on the counting sequence to 30. **M.CC-NCS.4.3** includes numbers 0-30. This asks for students to begin rote counting forward counting in a sequence from a number other than one (e.g. Given the number 4, the student would count, —4, 5, 6 . . ..). This objective does not require recognition of numerals. It is focused on the rote number sequence. Games that require students to add on to a previous count to reach a goal number encourage development of this concept. Frequent and brief opportunities utilizing counting on and counting back are recommended. These concepts emerge over time and cannot be forced. Examples: http://www.edplus.canterbury.ac.nz/literacy_numeracy/maths/numdocuments/dot_card_and_ten_frame_package2005.pdf
Common Misconceptions: Counting on or counting from a given number conflicts with the learned strategy of counting from the beginning. In order to be successful in counting on, students must understand cardinality (the number that ends the counting sequence represents how many objects are in the collection). Students often merge or separate two groups of objects and then re-count from the beginning to determine the final number of objects represented. For these students, counting is still a rote skill or the benefits of counting on have not been realized.

Domain: **Counting and Cardinality (CC)**

Cluster: Know number names and the count sequence.

Standard: **M.CC-NCS.4.2**
Represents a group of objects with a written numeral 0-12 (with 0 representing a count of no objects)

Standards for Mathematical Practice (SMP) to be emphasized:
MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:
This cluster is connected to the other clusters in the Counting and Cardinality Domain. For Preschool and Kindergarten to "Classify objects and count the number of objects in each category" for Kindergarten to "Add and subtract within 20 in Kindergarten" and for Grade 1 to "Extend the counting sequence".

Explanations and Examples:
M.CC-NCS.4.2 asks for students to represent a set of objects with a written numeral. The number of objects being recorded should not be greater than 12. Students can record the quantity of a set by selecting a number card/tile (numeral recognition) or writing the numeral. Students can also create a set of objects based on the numeral presented.

Students should be given multiple opportunities to count objects and recognize that a number represents a specific quantity. Once this is established, students begin to read and write numerals (numerals are the symbols for the quantities). The emphasis should first be on quantity and then connecting quantities to the written symbols.
A sample unit sequence might include:

1. Counting up to 12 objects in many settings and situations over several weeks.
2. Beginning to recognize, identify, and read the written numerals, and match the numerals to given sets of objects.
3. Writing the numerals to represent counted objects.

Since the teen numbers are not written as they are said, teaching the teen numbers as one group of ten and extra ones is foundational to understanding both the concept and the symbol that represents each teen number. For example, when focusing on the number 12, students should count out twelve objects using one-to-one correspondence and then use those objects to make one group of ten and two extra ones to begin to understand the representation to the symbol 12.

Common Misconceptions:
This standard addresses using the written numerals (0-12) to describe the amount of a set of objects. Due to varied development of fine motor and visual development, a reversal of numerals is anticipated for a majority of the students as they attempt to write them. While reversals should be pointed out to students, the emphasis is on the use of numerals to represent quantities rather than the correct handwriting

formation of the actual numeral itself.

Some students might not see zero as a number. Ask students to write 0 and say zero to represent the number of items left when all items have been taken away. Avoid using the word none to represent this situation.

Domain: **Counting and Cardinality (CC)**

Cluster: Count to tell the number of objects.

Standard: **M.CC-CNO.4.1**

Understand the relationship between numbers and quantities to 10; connect counting to cardinality.

a) Uses one to one correspondence when counting objects. Says the number names in the standard order pairing each object. (e.g., counts out napkins for snack time, saying the number aloud as they put each one on the table)

b)

1.Understands that the last number name said tells the number of objects counted (cardinality).

2. Understands that the number of objects remains the same regardless of the order in which they were counted.

c) Understand that each successive number name refers to a quantity that is one larger.

Standards for Mathematical Practice (SMP) to be emphasized:

MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the other clusters in the Counting and Cardinality Domain. For Preschool and Kindergarten to "Classify objects and count the number of objects in each category", and for Grade 1 to "Add and subtract within 20."

Explanations and Examples:

M.CC-CNO.4.1 asks students to count a set of objects and see sets and numerals in relationship to one another, rather than as isolated numbers or sets. These connections are higher-level skills that require students to analyze, to reason about, and to explain relationships between numbers and sets of objects. This standard should first be addressed using numbers 1-5 with teachers building to the numbers 1-10 later in the year. The expectation is that students are comfortable with these skills with the numbers 1-10 by the end of Kindergarten.

M.CC-CNO.4.1: a reflects the idea that students implement correct counting procedures by pointing to one object at a time (one-to-one correspondence) using one counting word for each object (one-to-one touching/synchrony), while keeping track of objects that have and have not been counted. This is the foundation of counting.

M.CC-CNO.4.1: b1 calls for students to answer the question- "How many are there?" by counting objects in a set and understanding that the last number stated when counting a set (…8, 9, 10) represents the total amount of objects: —"There are 10 bears in this pile" (cardinality). It also requires students to understand that the same set counted three different times will end up being the same amount each time.

M.CC-CNO.4.1: b2 is about the conservation of number, (regardless of the

arrangement of objects, the quantity remains the same); conservation of number is a developmental milestone, which even some Kindergarten students will not have mastered. The goal of this objective is for students to be able to count a set of objects; regardless of the formation those objects are placed.

M.CC-CNO.4.1 c represents the concept of "one more" while counting a set of objects. Students are to make the connection that if a set of objects was increased by one more object, the number name for that set is to be increased by one as well. Students are asked to understand this concept with and without objects. For example, after counting a set of 8 objects, students should be able to answer the question, — "How many would there be if we added one more object?; or answer a similar question when not using objects, by asking hypothetically, —"What if we have 5 cubes and added one more. How many cubes would there be then? " This concept should be first taught with numbers 1-5 before building to numbers 1-10.Students should be expected to be comfortable with this skill with numbers to 10 by the end of Kindergarten.

Domain: **Counting and Cardinality (CC)**

Cluster: Count to tell the number of objects.

Standard **M.CC-CNO.4.2**
Count to answer "how many" questions about as many as 10 things arranged in a line, a rectangular array, or a circle, or as many as 5 things in a scattered configuration.

Standards for Mathematical Practice (SMP) to be emphasized:
MP.2 Reason abstractly and quantitatively.
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Connections:
See Counting and Cardinality **M.CC-CNO.4.1** above

Explanations and Examples:
M.CC-CNO.4.2 addresses various counting strategies. From the research in early childhood mathematics, (Kathy Richardson), students go through a progression of four general ways to count. These counting strategies progress from least difficult to most difficult: 1) students move objects and count them as they move them, 2) students line up the objects and count them, 3) students have a scattered arrangement and they touch each object as they count and 4) students have a scattered arrangement and count them by visually scanning without touching them. Since the scattered arrangements are the most challenging for students, **M.CC-CNO.4.2** calls for students to only count 5 objects in a scattered arrangement, and count up to 10 objects in a line, rectangular array, or circle. Out of these 3 representations, a line is the easiest type of arrangement to count.

Students should develop counting strategies to help them organize the counting process to avoid re-counting or skipping objects.

Examples:
If items are placed in a circle, the student may mark or identify the starting object.
If items are in a scattered configuration, the student may move the objects into an organized pattern.
Counting up to 10 objects should be reinforced when collecting data to create charts and graphs.
Arizona and NC DOE

Common Misconceptions:
Some students might think that the count word used to tag an item is permanently connected to that item. So when the item is used again for counting and should be tagged with a different count word, the student may use the original count word. For example, a student counts 4 geometric figures: triangle, square, circle and rectangle with the count words: one, two, three, four. If these items are rearranged as rectangle, triangle, circle and square and counted, the student says these count words: four, one, three, two.

Number lines are not appropriate for pre-K, Kindergarten, or grade 1. See explanation and research in Flip Book 1st grade, pages 8 and 9. (NCTM, 2011). Instead use a number list (a list of numbers in order) or a number path (a list of numbers inside identical objects, such as squares) that allow students to play games and count along the list or path.

Domain: **Counting and Cardinality (CC)**

Cluster: Compare Numbers

Standard: **M.CC-CN.4.1**
Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group up to 10 by using matching and counting strategies. (e.g. compares the number of letters in their friend's names and indicates who has more and who has less.)

Standards for Mathematical Practice (SMP) to be emphasized:
MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Explanations and Examples:
M.CC-CN.4.1 Students can use matching strategies (student 1), counting strategies (student 2) or equal shares (student 3) to determine whether one group is greater than, less than, or equal to the number of objects in another group.

Student 1	Student 2	Student 3
"I lined up one square and one triangle. Since there is one extra triangle, there are more triangles than squares."	"I counted the squares and I got 8. Then, I counted the triangles and got 9. Since 9 is greater than 8, there are more triangles than squares."	"I put them in a pile. I then took away objects. Every time I took a square, I also took a triangle. When I had taken almost all of the shapes away, there was still a triangle left. That means that there are more triangles than squares."

As students develop meaning for numerals, they also compare these numerals to the quantities represented and their number words. Modeling numbers with manipulatives, such as dot cards and five- and ten-frames, serve as tools for such comparisons. Students can look for similarities and differences in these different representations of numbers. They begin to see the relationship of "one more", "one less", "two more" and "two less", leading to the concept that successive numbers name quantities where one is larger. In order to encourage this idea, students need discussion and reflection of pairs of numbers from 1 to 10. Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will greatly impact students's ability to break numbers into parts. Students demonstrate their understanding of the meaning of numbers when they can justify why their answer represents a quantity just counted. This justification could merely be the expression that the number said is the total because it was just counted, or a proof by demonstrating a one-to-one match, by counting again or other similar means (concretely or pictorially) that makes sense. An ultimate level of understanding is reached when

students can compare two numbers from 1 to10 represented as written numerals without counting.

Students need to explain their reasoning when they determine whether a number is "greater than", "less than", or "equal to" another number. Teachers need to ask probing questions such as "How do you know?" to elicit student's thinking. For students, these comparisons increase in difficulty, from "greater than" to "less than" to "equal". It is easier for students to identify differences than to find similarities. Students should develop a strong sense of the relationship between quantities and numerals before they begin comparing numbers.

Other strategies:
Matching: Students use one-to-one correspondence, repeatedly matching one object from one set with one object from the other set to determine which set has more objects.
Counting: Students count the objects in each set, and then identify which set has "more", "less", or an "equal" number of objects.
Observation: Students may use observation to compare two quantities (e.g., by looking at two sets of objects, they may be able to tell which set has "more" or "less" without counting).
Observations in comparing two quantities can be accomplished through daily routines of collecting and organizing data in displays. Students create object graphs and pictographs using data relevant to their lives (e.g., favorite ice cream, eye color, pets, etc.). Graphs may be constructed by groups of students as well as by individual students.
Benchmark Numbers: This would be the appropriate time to introduce the use of 0, 5 and 10 as benchmark numbers to help students further develop their sense of quantity as well as their ability to compare numbers.
Students state whether the number of objects in a set is "more", "less", or "equal" to a set that has 0, 5, or 10 objects.

Domain: **Counting and Cardinality (CC)**

Cluster: Compare numbers

Standard **M.CC-CN.4.3**
Compare two numbers between 1 and 5 when presented as written numerals. (e.g. 3 is more than 1; 4 is less than 5)

Standards for Mathematical Practice (SMP) to be emphasized:
MP. 2 Reason abstractly and quantitatively.
MP. 6 Attend to precision
MP. 7 Look for and make use of structure.
MP. 8 Look for and express regularity in repeated reasoning

Connections:
This cluster is connected to all clusters in the Operations and Algebraic Thinking Domain in Preschool, Kindergarten and Grade 1.

Explanations and Examples:
M.CC-CN.4.3 calls for students to apply their understanding of numerals 1-5 to compare one from the other. For example, looking at the numerals 3 and 5, a student must be able to recognize that the numeral 5 represents a larger quantity than the numeral 3. Students should begin this standard by having ample experiences with sets of objects (Standards 3 and 6) before completing this standard with just numerals. Students should not be introduced to this skill before the end of the preschool year.

Strategies: As students develop meaning for numerals, they also compare these numerals to the quantities represented and their number words. Modeling numbers with manipulatives, such as dot cards and five- and ten-frames, serve as tools for such comparisons. Students can look for similarities and differences in these different representations of numbers. They begin to see the relationship Between "one more", "one less", "two more" and "two less", thus landing on the concept that successive numbers name quantities where one is larger. In order to encourage this idea, students need discussion and reflection of pairs of numbers from 1 to 5. Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will impact students's ability to break numbers into parts.

Students demonstrate their understanding of the meaning of numbers when they can justify why their answer represents a quantity just counted. This justification could merely be the expression that the number said is the total because it was just counted, or a proof by demonstrating a one-to-one match, by counting again or other similar means (concretely or pictorially) that makes sense. An ultimate level of understanding is reached when students can compare two numbers from 1 to 5 represented as written numerals without counting.

Students need to explain their reasoning when they determine whether a number is "greater than", "less than", or "equal" to another number. Teachers need to ask probing questions such as " How do you know?" to elicit their thinking. For students,

these comparisons increase in difficulty, from “greater than” to “less than” to “equal”. It is easier for students to identify differences than to find similarities.

Arizona, Ohio, and NC DOE

Domain: **Counting and Cardinality**

Cluster: Compare Numbers

Standard **M.CC-CN.4.2**
Perceptually subitizes to 5 (instantly recognizes briefly shown collections up to 5 when presented in a variety of arrangements and verbally names the number of items.)

Standards for Mathematical Practice (SMP) to be emphasized:
MP. 2 Reason abstractly and quantitatively.
MP. 4 Model with mathematics.
MP. 7 Look for and make use of structure.

Explanations and Examples:
M.CC-CN.4.2 asks students to perceptually subitize groups to 5, which means that a student can look at a group of objects, or dots presented in a variety of arrangements and quickly name the number of items without counting them. Subitizing introduces basic ideas of cardinality or "how many"; as well as "more" and "less", parts and whole, and quantity. It is one of the main abilities young children should develop, yet seems to be lacking especially in children from low-resource communities and those with special needs.

Strategies:
To teach subitizing, place an arrangement of items (from 0-5), such as inch cubes, on a table during a small group and ask students to say the number of items on the table as fast as possible. Then reconfigure the items and repeat the activity. This can also be done using dots on a sheet of paper in various number and arrangements. Show the paper to the class and ask them to tell you the number as fast as possible.

When practicing subitizing groups, be sure to present items that are simple forms, such as circles or squares on a plain background which provides for high contrast. Do no use pictures or a mixture of shapes. Always start with linear arrangements (vertical, and horizontal), these are the easiest for young children, followed by rectangular arrangements (pairs of objects in rows, then move to "dice" or "domino" arrangements, leaving scrambled arrangements for last, as these are the most difficult.

Example of Arrangement types for Subitizing:

Easy	Medium	Difficult
□ □ □ □	□ □ □ □	□ □ □ □
□□□ □	□ □ □ □	□ □ □ □
□ □ □ □	□ □□□	

In addition to having students subitize one group of objects at a time, students can play matching games in which several cards or groups of items, all having the same number except for one are presented to the student. Ask the student, 'Which one does not belong?" Subitizing also includes rhythmic and spatial auditory patterns. In order to include these skills make a number of sounds (e.g. 3 claps or 5 rings of a bell) and ask students to either write the number they heard or hold up the corresponding number of fingers.

Common Misconceptions:
Students will sometimes try to "guess" how many there are instead of developing strategies to "see" the patterns and the numbers. In order to combat this, stop and allow students to share their strategies of how they came to the conclusion of the number within the group of items without counting. If students are guessing, they are not using mathematical thinking and therefore the activity may not be a good use of instructional time (Clements and Sarama, 2009).

Domain**: Operations and Algebraic Thinking (OA)**

Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. All standards in this cluster should only include numbers through 5. Students will model simple joining and separating situations with sets of objects, drawings or acting out situations. (Preschool students should see addition and subtraction equations. Student writing of equations is not required). Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

Standard: **M.OA-AS.4.1**

Demonstrate an understanding of addition and subtraction with objects, fingers, and drawings (drawings need not show details, but should show the mathematics in the problems. This applies wherever drawings are mentioned in the Standards.) and acting out practical situations (e.g. if we have 3 toy giraffes in the block area and we add 2 toy elephants, how many animals will be have all together?)

Standards for Mathematical Practice (SMP) to be emphasized:

MP. 1 Make sense of problems and persevere in solving them.
MP. 2 Reason abstractly and quantitatively.
MP. 4 Model with mathematics.
MP. 5 Use appropriate tools strategically.

Explanations and Examples:

M.OA-AS.4.1 asks students to demonstrate the understanding of how objects can be joined (addition) and separated (subtraction) by representing addition and subtraction situations in various ways. This objective is primarily focused on understanding the concept of addition and subtraction, rather than merely reading and solving addition and subtraction number sentences (equations).

Instructional Strategies: Using addition and subtraction in a word problem context allows students to develop their understanding of what it means to add and subtract.

Students should use objects, fingers drawing, and acting out situations in order to develop the concepts of addition and subtraction. Then, they should be introduced to written expressions and equations using appropriate terminology and symbols which include +, –, and =.

Addition terminology: add, join, put together, plus, combine, total

Subtraction terminology: minus, take away, separate, difference, compare

Have students decompose numbers less than or equal to 5 during a variety of experiences to promote their fluency with sums and differences less than or equal to 5 that result from using the numbers 0 to 5. For example, ask students to use different models to decompose numbers to 5 and record their work with drawings. As students begin to understand the role and meaning of arithmetic operations in number systems, they will gain computational fluency, and how to use efficient and

accurate methods for computing. The teacher can use back mapping and scaffolding to teach students who show a need for more help with counting. For example, ask students to build a tower of 3 using 1 green and 2 blue linking cubes while you discuss composing and decomposing 3. Have them identify and compare other ways to make a tower of 2. Repeat the activity for towers of 4 and 5. Help students use counting as they explore ways to compose numbers 3-5

Common Misconceptions:

Students may overgeneralize the vocabulary in word problems and think that certain words indicate solution strategies that must be used to find an answer. They might think that the word "more" always means to add and the words "take away" or "left" always means to subtract. When students use the words "take away" to refer to subtraction and its symbol, teachers need to repeat students' ideas using the words "minus" or "subtract". For example, students use addition to solve this "Take from/Start Unknown" problem: Melisa took the 2 stickers she no longer wanted and gave them to Anna. Now Melisa has 3 stickers left. How many stickers did Melisa have to begin with?

Note on vocabulary: The term total should be used instead of the term sum. Sum sounds the same as some, but has the opposite meaning. Some is used to describe problem situations with one or both addends unknown, so it is better in the earlier grades to use total rather than sum. Formal vocabulary for subtraction (minuend and subtrahend) is not needed in Preschool. (total and add are sufficient for classroom discussions).

Students should be encouraged to use create drawings /pictorial representations of the problems and/or situation.

If students progress from working with manipulatives to writing numerical expressions and equations, and they skip using pictorial thinking—students will then be more likely to use finger counting and rote memorization for work with addition and subtraction.

Counting forward builds to the concept of addition while counting back leads to the concept of subtraction. However, counting is an inefficient strategy. Teachers need to provide instructional experiences so that students progress from the concrete level, to the pictorial level, then to the abstract level when learning mathematical concepts. (Concrete, Representational, Abstract CRA)

Just knowing the basic facts is not enough. We need to help students develop the ability to quickly and accurately understand the relationships between numbers. They need to make sense of numbers as they find and make strategies for joining and separating quantities

Arizona and NC DOE

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Standard **M.OA-AS.4.2**
Compose and decompose numbers less than or equal to 5 into pairs in more than one way (i.e. by using objects).

Standards for Mathematical Practice (SMP) to be emphasized:
MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.4 Model with mathematics.
MP.6 Attend to precision
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Explanations and Examples:
M.OA-AS.4.2 asks students to understand that a set of (5) objects can be broken into two sets (3 and 2) and still be the same total amount (5). The focus is on number pairs, which add to a specified total, 1-5. This standard also asks students to understand that a set of objects (5) can be broken in multiple ways (3 and 2; 4 and 1). Thus, when breaking apart a set (decomposing), students develop the understanding that a smaller set of objects exists within that larger set (inclusion). This should be developed in context before moving into how to represent decomposition with symbols (+, -, =).

Example:
—Bobby Bear is missing 5 buttons on his jacket. How many ways can you use blue and red buttons to finish his jacket? Use blue and red lacing links to show your options.
Students could make towers of:
4 blue and 1 red cubes
3 blue and 2 red cubes
2 blue and 3 red cubes
1 blue and 4 red cubes

After the students have had numerous experiences with decomposing sets of objects and recording with pictures, the teacher eventually makes connections between the drawings and symbols: 5=4+1, 5=3+2, 5=2+3, and 5=1+4. The number sentence only comes after pictures or work with manipulatives, and students should never give the number sentence without a mathematical representation. Students may use objects such as cubes, two-color counters, square tiles, etc. to show different number pairs for a given number. For example, for the number 5, students may split a set of 5 objects into 1 and 4, 2 and 3, etc.

Arizona, Ohio, and NC DOE

Domain: **Operations and Algebraic Thinking**

Cluster: Understanding addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Standard **M.OA-AS.4.3**
Identifies patterns in the real world and in numbers (e.g. +1 pattern where one more than 3 is 4, one more than 4 is 5, written numerals follow a definite number pattern as the digits repeat, 100's chart).

Standards for Mathematical Practices (SMP) to be emphasized:
MP. 4 Model with mathematics
MP. 6 Attend to precision
MP. 7 Look for and make use of structure
MP. 8 Look for and express regularity in repeated reasoning

Explanations and Examples:
M.OA-AS.4.3 calls for students to identify patterns in the real world including repeating units of shapes, colors, sounds, and movements; as well as identifying the patterns in our number system. Counting involves patterns beyond simple repeating patterns. The list of counting words has an intricate pattern that repeats the number 0-9 in the ones, tens, hundreds, etc. places. While students in early childhood will not fully understand this pattern until later in elementary school, the foundation for this pattern should be laid in preschool as students identify and use the repeating patterns in the number words to 100. This includes developing an understanding that adding "one more" to a number is always the next number in the sequence, and seeing the repetition in our written numeral system. Identifying patterns helps to bring order and predictability to seemingly unorganized situations and allows students to make new generalizations. In the early years, recognition and analysis of patterns provides the foundation for algebraic thinking. An important piece of this is helping students identify a unit (e.g. AB) that will repeat (ABABABAB) and then use that information to create another pattern.

Instructional Strategies
It is important for students to recognize patterns in both the world and numbers. Help students attend to patterns around them and play movement games involving patterns. Take students on pattern walks to find and discuss the patterns they see. Find patterns in students' clothing and in combinations of movements. Discuss the patterns present in numbers as digits repeat. Start patterns with manipulatives and encourage students to identify the pattern and extend it.

Common Misconceptions:
Recognizing patterns and organizing information are a part of recognizing structure, which is important at all levels of mathematics, creating a foundation for later algebraic thinking. While activities involving patterns are appropriate and important in early childhood, they form a small part of the mathematics content for early childhood and should be balanced with a greater emphasis put on numbers and operations, geometry and measurement.

Domain: **Measure and Data (MD)**

Cluster: Describe and compare measureable attributes.

Standard: **M.MD-MA.4.1**
Describe and compare two objects using measureable attributes (length, size, capacity, and weight).

Standards for Mathematical Practice (SMP) to be emphasized:
MP.4 Model with mathematics
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure

Explanations and Examples:
M.MD-MA.4.1 calls for students to describe measurable attributes of objects, such as length and weight. In order to describe attributes, such as length and weight, students must have many opportunities to informally explore these attributes.
Students should state comparisons of objects verbally and then focus on specific attributes when making verbal comparisons. They may identify measurable attributes such as length, width, height, and weight. For example, when describing a soda can, a student may talk about "how tall", "how wide", "how heavy", or "how much liquid can fit inside". These are all measurable attributes. Non-measurable attributes include: words about object, colors, pictures, etc.

This standard focuses on using descriptive words and does not mean that students should sort objects based on attributes. (Sorting appears later).

Instructional Strategies:
It is critical for students to be able to identify and describe measureable attributes of objects. An object has different attributes that can be measured, like the height and weight of a can of food.
Students should be given many opportunities to compare directly where the attribute becomes the focus. For example, when comparing the volume of two different boxes, ask students to discuss and justify their answers to these questions: "Which box will hold the most?", "Which box will hold the least?", "Will they hold the same amount?" "How could you find out?" Students can decide to fill one box with dried beans, and then pour the beans into the other box to determine the answers to these questions.

Have students work in pairs to compare their arm spans. As they stand back-to-back with outstretched arms, compare the lengths of their spans, and then determine who has the shortest arm span. Ask students to explain their reasoning. Then ask students to suggest other measureable attributes of their bodies that they could directly compare, such as their height or the length of their feet.

Domain: **Measurement and Data (MD)**

Cluster: Describe and compare measureable attributes.

Standard: **M.MD-MA.4.2**
Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. (e.g. compare the heights of two children and describe one child as taller or shorter).

Standards for Mathematical Practice (SMP) to be emphasized:
MP.2 Reason abstractly and quantitatively
MP.4 Model with mathematics
MP.6 Attend to precision
MP.7 Look for and make use of structure.

Explanations and Examples:
M.MD-MA.4.2 asks for direct comparisons of objects. Direct comparisons are made when objects are put next to each other, such as two children, two books, two pencils. For example, a student may line up two blocks and say, —"This block is a lot longer than this one".

Students are not comparing objects that cannot be moved and lined up next to each other. When making direct comparisons for length, students must attend to the starting point of each object and recognize that objects should be matched up at the end of objects to get accurate measurements. For example, the ends need to be lined up at the same point, or students need to compensate when the starting points are not lined up (conservation of length includes understanding that if an object is moved, its length does not change; an important concept when comparing the lengths of two objects). Since this understanding requires conservation of length, a developmental milestone for young children, students need multiple experiences to move beyond the idea that "Sometimes this block is longer than this one and sometimes it's shorter (depending on how I lay them side by side) and that's okay." "This block is always longer than this block (with each end lined up appropriately)." Before conservation of length: The blue block is longer than the plain block when they are lined up like this. But when I move the blocks around, sometimes the plain block is longer than the blue block. After conservation of length: I have to line up the blocks to measure them.

Language plays an important role in this standard as students describe the similarities and differences of measurable attributes of objects (e.g., "shorter than", "taller than", "lighter than", "the same as").
Students should have many opportunities to compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object).

Common Misconceptions:
Many students have difficulty understanding that when an object is moved away from another object they are comparing it with the length does not change. With multiple opportunities, students learn that they have to line up the items they are comparing and/or measuring. (Conservation of Length: includes understanding that

if an object is moved, its length does not change; an important concept when comparing the lengths of two objects).

Arizona, Ohio, and NC DOE

Domain: **Measurement and Data**
Cluster: Describe and compare measurable attributes
Standard **M.MD-MA.4.3** Measure length and volume (capacity) using non-standard or standard measurement tools.
Standards for Mathematical Practice (SMP) to be emphasized: MP 2 Reason abstractly and quantitatively MP 3 Construct viable arguments and critique the reasoning of others MP 5 Use appropriate tools strategically MP 7 Look for and make use of structure
Explanations and Examples: **M.MD-MA.4.3** asks students to determine either the length of an object or the capacity of a container using standard and non- standard tools, in order to solidify unit iteration. Unit Iteration is an understanding that the length or capacity of a smaller item is part of the overall length or capacity of the object being measured; and to measure the object, one must place the smaller unit repeatedly along the length of the larger object. A student playing in the water table may use one container (a nonstandard measure) to fill up another and proclaim, "It took 4 containers to fill it up". At the same water table, another student might use measuring cups to fill a container and determine that container holds 4 cups (a standard measure). When measuring length, a student must identify a unit of measure and subdivide the object by that unit, placing that unit end to end along the object. This is best done with both conventional rulers and manipulative units, beginning the process of actual measurement. **Instructional Examples:** Teachers may introduce the relationships between the size and number of length units needed to measure a particular object. Classroom discussions should focus on the answer to, "What are you counting?" being length units. Teachers also will want introduce the relationships between the size and number of length units. They can do this by encouraging the comparisons of the results of measuring the same object with varying manipulatives. For example, when measuring the length of a table, have one student measure using inch cubes while another measures the same table using a 5-inch string. Discuss the results and the students' conclusions about why they came up with different number amounts in their measurement.
Common Misperceptions Initially, some students may not recognize the need for equal length units and see no problem mixing units (e.g. using both paperclips and pen caps) as long as they cover the entire unit in some way. In order to support their learning, it is best to use consistent units of equal length to develop the idea and need for equal length units.

If the units are not of equal length, then they are not really a unit that can be used for measuring. When working with students as they count units it is important to help them label the unit of measurement (e.g. "I used 7 blocks to measure the flag."). Students may also make errors by leaving gaps between the units instead of aligning them end-to-end. Teachers can help students develop the understanding of the necessity to align objects without gaps by comparing a student's results with the results of another student who didn't have gaps and discussing the reasons for the varying results.

Engaging Young Children in Mathematics, 2004; Learning and Teaching Early Math, 2009, PreK Curriculum Focal Points

Domain: **Measurement and Data (MD)**

Cluster: Classify objects and count the number of objects in each category.

Standard: **M.MD-CCC.4.1**
Sort objects into categories; count the number of objects in each category (limit category counts to 10 or fewer) and make comparisons between the categories based on quantity.

Standards for Mathematical Practice (SMP) to be emphasized:
MP.2 Reason abstractly and quantitatively
MP.6 Attend to precision
MP.7 Look for and make use of structure.

Connections:
This cluster is connected to Know number names and the count sequence and Count to tell the number of objects for Preschool and Kindergarten, and to Represent and interpret data for Grade 1.

Explanations and Examples:
M.MD-CCC.4.1 asks students to identify similarities and differences between objects (e.g., size, color, shape) and use the identified attributes to sort a collection of objects. Once the objects are sorted, the student counts the amount in each set. Once each set is counted, then the student is asked to sort (or group) each of the sets by the amount in each set.
For example, when given a collection of buttons, the student separates the buttons into different piles based on color (all the blue buttons are in one pile, all the orange buttons are in a different pile, etc.). Then the student counts the number of buttons in each pile: blue (5), green (4), orange (3), purple (4). Finally, the student organizes the groups by the quantity in each group (Orange buttons (3), Green buttons (4), Purple buttons with the green buttons because purple also had (4), Blue buttons last (5).

Other possible objects to sort include: shells, shapes, beans, small toys, coins, rocks, etc. After sorting and counting, it is important for students to:

- explain how they sorted the objects;
- label each set with a category;
- answer a variety of counting questions that ask, "*How many*"; and
- compare sorted groups using words such as, "*most*", "*least*", "*alike*" and "*different*".

This objective helps to build a foundation for data collection in future grades. In later grades, students will transfer these skills to creating and analyzing various graphical representations.

Instructional Strategies:
Provide categories for students to use to sort a collection of objects. Each category can relate to only one attribute, like "Red" and "Not Red" or "Hexagon" and "Not Hexagon", and contain up to 10 objects. Students count how many objects are in each category and then order the categories by the number of objects they contain.

Ask questions to initiate discussion about the attributes of shapes. Then have students sort a collection of two-dimensional and three-dimensional shapes by their attributes. Provide categories like “Circles” and “Not Circles” or “Flat” and “Not Flat”. Have students count the objects in each category and order the categories by the number of objects they contain. Have students infer the classification of objects by guessing the rule for a sort. First, the teacher uses one attribute to sort objects into two loops or regions without labels. Then the students determine how the objects were sorted, suggest labels for the two categories and explain their reasoning.

Arizona, Ohio, and NC DOE

Domain: **Measurement and Data (MD)**

Cluster: Classify objects and count the number of objects in each category.

Standard: **M.MD-CCC.4.2**
Collect data by categories to answer simple questions.

Standards for Mathematical Practice (SMP) to be emphasized:
MP.2 Reason abstractly and quantitatively
MP.6 Attend to precision
MP.7 Look for and make use of structure.

Connections:
This cluster is connected to Know number names and the count sequence and Count to tell the number of objects for Preschool and Kindergarten, and to Represent and interpret data for Grade 1.

Explanations and Examples:
M.MD-CCC.4.2 asks students to identify similarities and differences between objects (e.g., size, color, shape) and use the identified attributes to sort a collection of objects. Once the objects are sorted, the student counts the amount in each set. Once each set is counted, then the student is asked to sort (or group) each of the sets by the amount in each set.

For example, when given a collection of buttons, the student separates the buttons into different piles based on color (all the blue buttons are in one pile, all the orange buttons are in a different pile, etc.). Then the student counts the number of buttons in each pile: blue (5), green (4), orange (3), purple (4). Finally, the student organizes the groups by the quantity in each group (Orange buttons (3), Green buttons (4), Purple buttons with the green buttons because purple also had (4), Blue buttons last (5)).

Other possible objects to sort include: shells, shapes, beans, small toys, coins, rocks, etc. After sorting and counting, it is important for students to:

- *explain how they sorted the objects;*
- *label each set with a category;*
- *answer a variety of counting questions that ask, "How many …"; and*
- *compare sorted groups using words such as, "most", "least", "alike" and "different".*

This helps to build a foundation for data collection in future grades. In later grades, students will transfer these skills to creating and analyzing various graphical representations.

Instructional Strategies:
Provide categories for students to use to sort a collection of objects. Each category can relate to only one attribute, like "*Red"* and "*Not Red"* or "*Square"* and "*Not Square",* and contain up to 10 objects. Students count how many objects are in each category and then order the categories by the number of objects they contain.

Ask questions to initiate discussion about the attributes of shapes, then have students sort a collection of two-dimensional and three-dimensional shapes by their attributes. Provide categories like "*Circles"* and "*Not Circles"* or "*Flat"* and "*Not Flat"*. Have students count the objects in each category and order the categories by the number of objects they contain.

Have students infer the classification of objects by guessing the rule for a sort. First, the teacher uses one attribute to sort objects into two loops or regions without labels. Then the students determine how the objects were sorted, suggest labels for the two categories and explain their reasoning.

Arizona, Ohio & NC DOE:

Domain: **Geometry (G)**

Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, cubes, cones, cylinders, and spheres).

Standard: **M.G-IDS.4.1**
Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as "above", "below", "beside", "in front of", "behind", and "next to".

Standards for Mathematical Practice (SMP) to be emphasized:
MP.6 Attend to precision
MP.7 Look for and make use of structure.

Connections:
This cluster is connected to *Analyze, compare, create and compose shapes* for Preschool and Kindergarten, and to *Reason with shapes and their attributes* for Grade 1.

Explanations and Examples:
M.G-IDS.4.1 expects students to use positional words (such as those italicized above) to describe objects in the environment. Preschool students need to focus first on location and position of two-and three-dimensional objects in their classroom prior to describing location and position of two-and three-dimension representations on paper. Examples of environments in which students would be encouraged to identify shapes would include nature, buildings, home and the classroom using positional words in their descriptions. Teachers should work with students and pose four mathematical questions:

- *"Which way?"*
- *"How far?"*
- *"Where?"*
- *"What objects?"*

To answer these questions, students develop a variety of important skills contributing to their spatial thinking.

Examples:

- Teacher holds up an object such as an ice cream cone, a number cube, ball, etc. and asks students to identify the shape. Teacher holds up a can of soup and asks, "*What shape is this can?"* Students respond —"*cylinder!*"
- Teacher places an object "next to," "behind", "above", "below", "beside", or "in front" of another object and asks positional questions. "*Where is the water bottle?*" (water bottle is placed behind a book) Students say —"*The water bottle is behind the book."*

Students should have multiple opportunities to identify shapes; these may be displayed as photographs, or pictures using the document camera or interactive whiteboard.

Instructional Strategies:
Develop spatial sense by connecting geometric shapes to students' everyday lives. Initiate natural conversations about shapes in the environment. Have students

identify and name two- and three-dimensional shapes in and outside of the classroom and describe their relative position. Ask students to find rectangles in the classroom and describe the relative positions of the rectangles they see (e.g. "*This rectangle (a poster) is over the sphere or globe*.") Teachers can use a digital camera to record these relationships.

Hide shapes around the room. Have students say where they found the shape using positional words (e.g. "*I found a triangle UNDER the chair.*")

Have students create drawings involving shapes and positional words: "*Draw a window "on" the door*", or "*Draw an apple "under" a tree*". Some students may be able to follow two- or three-step instructions to create their drawings.

Use a shape in different orientations and sizes along with non-examples of the shape so students can learn to focus on defining attributes of the shape.

Manipulatives used for shape identification actually have three dimensions. However, preschoolers need to think of these shapes as two-dimensional or "flat" and typical three-dimensional shapes as "solid". Students will identify two-dimensional shapes that form surfaces on three-dimensional objects. Students need to focus on noticing two and three dimensions, not on the words two-dimensional and three-dimensional. You may create an activity where an object is identified and whispered into the teacher's ear. Students then ask questions of the first student---"*is it in front of*", etc.

Common Misconception:
Students many times use incorrect terminology when describing shapes. For example, students may say a cube is a square or that a sphere is a circle. The use of the two-dimensional shape that appears to be part of a three-dimensional shape to name the three-dimensional shape is a common misconception. Work with students to help them understand that the two-dimensional shape is a part of the object but it has a different name.

Arizona, Ohio and NC DOE

Domain: **Geometry (G)**

Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, cubes, cones, cylinders, and spheres).

Standard: **M.G-IDS.4.2**.
Correctly name shapes regardless of their orientations or overall size

Standards for Mathematical Practice (SMP) to be emphasized:
MP.6 Attend to precision
MP.7 Look for and make use of structure.

Connections:
This cluster is connected to *Analyze, compare, create and compose shapes* for Preschool and Kindergarten, and to *Reason with shapes and their attributes* for Grade 1.

Explanations and Examples:
M.G-IDS.4.2 addresses students' identification of shapes based on known examples. Students at this level do not yet recognize triangles that are turned upside down as triangles, since they do not look like triangles. Students need many experiences looking at and manipulating shapes with various typical and atypical orientations. Through these experiences, students will begin to move beyond what a shape looks like to identifying particular geometric attributes that define a shape. Students should be exposed to many types of triangles in many different orientations in order to eliminate the misconception that a triangle is always right-side-up and equilateral.
Students should also be exposed to many shapes in many different sizes.

Examples:

- Teacher makes pairs of paper shapes that are different sizes. Each student is given one shape and the objective is to find the partner who has the same shape.
- Teacher brings in a variety of spheres (tennis ball, basketball, globe, ping pong ball, etc.) to demonstrate that size doesn't change the name of a shape.

Instructional Strategies:
Preschoolers form visual templates, or models of shape categories. For example, students recognize a shape as rectangle because it looks like a door. Because students base their understanding of shapes on examples, they need to experience a rich variety of shapes in each shape category so that their mental models are not overly restricted. Students without good experiences often reject both triangles and rectangles that are too "skinny" or "not wide enough" Students should see examples of rectangles that are long and skinny, and they should contrast rectangles with non-rectangles that appear similar but do not have an important defining attribute. Similarly, they should see examples of triangles that have sides of three different lengths, and they should contrast triangles with non-triangles.

Students also need to see examples of shapes beyond circles, squares, rectangles, and triangles. Without these, students develop limited notions. (i.e. many children

come to believe incorrectly that a trapezoid is not a shape because it is not a shape for which they know a name).

Preschoolers should also learn to recognize these shapes whether they are in standard position or rotated so that their bases are not horizontal.

Preschoolers can begin to develop explicit and sophisticated levels of thinking and communication. They can learn to describe, and even define, shapes in terms of their parts or attributes (properties). For example, they can build accurate representations of shapes from physical models of line segments, such as sticks. As they discuss what they have built, attributes of the shapes will arise naturally.

Example:
Student: "*I built a rectangle."*
Teacher: "*How do you know it is a rectangle?"*
Student: "*Because these two sides are the same length."*

The experience of discussing attributes of rectangles (or any shape they build) helps students begin to understand the geometric structure of all rectangles at an explicit level of thinking.

Another valuable activity is the tactile-kinesthetic exploration of shapes—feeling shapes hidden in a box. Preschoolers can name the shape they are feeling rather than just match shapes. After this, they can extend the activity further as they describe the shape without using its name, so that their friends can name the shape. In this way, students learn the properties of the shape, moving from intuitive to explicit, verbalized knowledge. All these variations can be repeated with less familiar shapes.

Such activities help students learn to identify and describe shapes by the number of their sides or corners. Such descriptions build geometric concepts but also reasoning skills and language. They encourage students to view shapes analytically. Students begin to describe some shapes in terms of their properties, such as saying that squares have four side of equal length. They informally describe properties of blocks in functional contexts, such as that some blocks roll and other do not.

Focus In PreKindergarten, NCTM 2010

Domain: **Geometry (G)**

Cluster: Analyze, compare, create, and compose shapes.

Standard: **M.G-ACCC.4.1**
Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

Standards for Mathematical Practice (SMP) to be emphasized:
MP.4 Model with mathematics
MP.6 Attend to precision
MP.7 Look for and make use of structure.

Connections:
This cluster is connected to *Identify and describe shapes (squares, circles, triangles, rectangles, hexagon, cubes, cones, cylinders, and spheres)* for Preschool and Kindergarten, and to *Reason with shapes and their attributes* for Grade 1.

Explanations and Examples:
M.G-ACCC.4.1 asks students to note similarities and differences between and among 2 dimensional and 3 dimensional shapes using informal language. These experiences help young students begin to understand how 3-dimensional shapes are composed of 2-dimensional shapes (e.g.. The base and the top of a cylinder is a circle; a circle is formed when tracing a sphere).

Students analyze and compare two- and three-dimensional shapes by observations. Their visual thinking enables them to determine if things are alike or different based on the appearance of the shape. Students sort objects based on appearance. Even in early explorations of geometric properties, they are introduced to how categories of shapes are subsume (contained) within other categories. For instance, they will recognize that a square is a special type of rectangle.

Students should be exposed to triangles, rectangles, and hexagons whose sides are not all congruent. They first begin to describe these shapes using everyday language and then refine their vocabulary to include sides and vertices/corners. Opportunities to work with pictorial representations, concrete objects, as well as technology helps student develop their understanding and descriptive vocabulary for both two- and three- dimensional shapes.

Instructional Strategies:
The abilities involved in composing and decomposing shapes are important for many reasons. These geometric competencies are at the foundation of geometry, but also arithmetic (e.g., composing and decomposing numbers and arrays in multiplication), measurement, and higher order geometric work. Creating and then iterating units and higher-order units in the context of construction patterns, measuring, and computing, are established bases for mathematical understanding and analysis.

It is important to allow students to explore and build geometric understanding themselves. One important step to take is to switch from making assertions and generalizations to framing ideas as questions. Rather than saying, "*Every time you put two triangles together, you get a square*", a mathematically incorrect statement. Ask the following: "*How many different ways can you put these two triangles together to make a new shape*?" or "*What shapes will you get?"*

This allows students to see that even with two right triangles made from a square, they can put these together to make a triangle or a parallelogram.

Preschoolers can develop the ability to intentionally and systematically combine shapes to make new shapes and complete puzzles. They do so with increasing anticipation, on the basis of the shapes' attributes, and thus, students developmental imagery of the component shapes. They move from using shapes separately to putting them together to make pictures.

A significant advance is that they can combine shapes with different properties, extending the pattern block shapes (whose angles are multiples of 30 degrees) common at early levels to such shapes as tangrams (with angles that are multiples of 45 degrees), and with sets of various shapes that include angles that are multiples of 15 degrees, as well as sections of circles. Combining these shape sets should be done after students have worked with the pattern-block shapes separately from the square/rectangle/right triangle shapes based on 90 degrees and 45 degrees because many compositions are possible when the angles are consistent (Focus in PreKindergarten, NCTM 2010).

Use shapes collected from students to begin the investigation into basic properties and characteristics of two- and three-dimensional shapes. Have students analyze and compare each shape with other objects in the classroom and describe the similarities and differences between the shapes. Ask students to describe the shapes while the teacher records key descriptive words in common student language. Students may use the word "flat" to describe two-dimensional shapes, and the word "solid" to describe three-dimensional shapes.

Use the sides, faces and vertices of shapes to practice counting and reinforce the concept of one-to-one correspondence.

The teacher and students orally describe and name the shapes found on a "Shape Hunt". Students draw a shape and build it using materials regularly kept in the classroom such as construction paper, clay, wooden sticks or straws.

Students can use a variety of manipulatives and real-world objects to build larger shapes with these and other smaller shapes: squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. Preschoolers can manipulate cardboard shapes, paper plates, pattern blocks, tiles, canned food, and other

common items.

Have students compose (build) a larger shape using only smaller shapes that have the same size and shape. The sides of the smaller shapes should touch and there should be no gaps or overlaps within the larger shape. For example, use one-inch squares to build a larger square with no gaps or overlaps. Have students also use different shapes to form a larger shape where the sides of the smaller shapes are touching and there are no gaps or overlaps. Ask students to describe the larger shape and the shapes that formed it.

Common Misconceptions:
One of the most common misconceptions in geometry is the belief that orientation is tied to shape. A student may see a triangle, but claim to not know the name of another triangle that is turned on its side.

Another misconception is confusing the name of a two-dimensional shape with a related three-dimensional shape or the shape of its face. For example, students might call a cube a square because the student sees the face of the cube.
It is important when students are exploring 2-dimensional shapes (flat) that the shapes they are working with are on paper or other —"FLAT" material.

Arizona, Ohio & NC DOE

Domain: **Geometry (G)**

Cluster: Analyze, compare, create, and compose shapes.

Standard: **M.G-ACCC.4.2**
Create shapes during play by building or drawing.

Standards for Mathematical Practice (SMP) to be emphasized:
MP.1 Make sense of problems and persevere in solving them.
MP.4 Model with mathematics.
MP.7 Look for and make use of structure.

Connections:
This cluster is connected to Preschool and Kindergarten, *Describing shapes and space*. It is also connected to *Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)* for Preschool and Kindergarten, and to *Reason with shapes and their attributes* for Grade 1.

Explanations and Examples:
M.G-ACCC.4.2 asks students to apply their understanding of geometric attributes of shapes in order to create given shapes. . Students often do not think about shapes in terms of their parts or attributes. To help preschoolers begin this kind of thinking, it is important for them to build and construct a variety of shapes. It is a challenge for students to integrate the parts of a shape into a whole, but construction of them helps solidify this understanding and develops a mature understanding of shapes and their defining attributes. For example, a student may roll a clump of play-doh into a sphere or use their finger to draw a triangle in the sand table, recalling various attributes in order to create that particular shape. While the shape the student makes may not be the most accurate, the point of the activity is to get the students thinking about what must be done to build the shape.

Instructional Strategies:
Students might make squares, rectangles or triangle from different lengths of sticks, with their bodies, or pair with a friend to make a shape. Three-dimensional shapes may be built using materials such as clay, toothpicks, marshmallows, gumdrops, straws, pipe cleaners, etc.

Focus in Pre Kindergarten, NCTM 2010

Domain: **Geometry (G)**

Cluster: Analyze, compare, create, and compose shapes.

Standard: **M.G-ACCC.4.3**
Put together several shapes to make a picture and fill simple outline puzzles.

Standards for Mathematical Practice (SMP) to be emphasized:
MP.1 Make sense of problems and persevere in solving them.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.7 Look for and make use of structure.

Connections:
This cluster is connected to Preschool and Kindergarten, *Describing shapes and space*. It is also connected to *Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)* for Preschool and Kindergarten, and to *Reason with shapes and their attributes* for Grade 1.

Explanations and Examples:
M.G-ACCC.4.3 moves beyond identifying and classifying simple shapes to manipulating two or more shapes to create a new shape. This concept begins to develop as students first move, rotate, flip, and arrange puzzle pieces. Next, students use their experiences with puzzles to move given shapes to make a design (e.g., Use the 7 tangram pieces to make a fox.). Finally, using these previous foundational experiences, students manipulate simple shapes to make a new shape.

Instructional Strategies:
Students use pattern blocks, tiles, or paper shapes and technology to make new two- and three-dimensional shapes. Their investigations allow them to determine what kinds of shapes they can join to create new shapes. They answer questions such as "*What shapes can you use to make a square, rectangle, circle, triangle?*"

This is an opportunity to use blocks from a play center to create shapes composed of a series of blocks. Laying several rectangular prisms can make other identifiable shapes.

Students may use a document camera to display shapes they have composed from other shapes. They may also use an interactive whiteboard to copy shapes and compose new shapes. They should describe and name the new shape.

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