Computation of Fractions: Part III

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Overview Part III:

- Overview of Fractions
- Party Flags Findings
- National Mathematics Advisory Panel (NMAP)
  - Instruction Recommendations
- Institute of Education Science (IES)
  - Instructional recommendations
- Important Fraction Concepts
  - Equivalent Fractions
  - Simplifying Fractions
  - Computation with Fractions
- Conclusions

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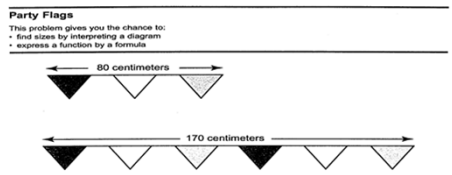

Erica is putting up lines of colored flags for a party.

The flags are all the same size and are spaced equally along the line.

1. Calculate the length of the sides of each flag, and the space between flags.

Show all your work clearly.

** From David Foster
Silicon Valley Math Initiative
Algebra Forum Online Event 10/23

2. How long will a line of *n* flags be?

Write down a formula to show how long a line of *n* flags would be.

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Poll: How did you solve question #1 of Party Flags?

A. I looked at one set of flags, used guess and check, and figured the flag length was 20 and the string 10.

B. I looked and both sets of flags and used a process where I double the first set of flags, compared it to the second set of flags ~~and found that the string must be 10. Then I was~~ able to go back and find the flag length was 20.

C. I set up two equations with two unknowns and solve it using simultaneous equations.

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The Findings from Party Flags

- The task may be approached as a system of simultaneous equations, almost no algebra students used such an approach.
- 49% of algebra students had no success.
- 44% accurately found the two lengths (most commonly by an estimation strategy only using one constraint).
- 21% correctly used both constraints (the length of three flags is 80 cm. and the length of 6 flags is 170 cm.).
- 7% of the students were able to develop a valid generalization for n flags.

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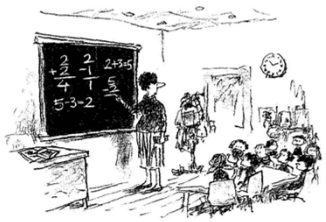
**MAC Final Report 2001



"Please, Ms. Sweeney, may I ask where you're going with all this?"

Students who spend too much energy with basic computation have a difficult time learning the concepts of secondary math.

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National Math Panel quotes: K-8th grades

- "Difficulty with fractions (including decimals and percents) is pervasive and is a major obstacle to further progress in mathematics, including algebra"
- Tom Loveless stated, "Students don't know how to translate fractions into decimals or into percentages and they can't locate fractions on a number line"

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Do middle school math courses add up to algebra preparation?

Sanders, Riccomini, & Witzel, 2005

Code	Category	Entering Math Tech 1	Entering Algebra 1
DAPR	Data Analysis, Probability & Statistics	39 (46.4%)	85 (88.5%)
DECM	Decimals, their Operations and Applications: Percent	11 (13.1%)	64 (66.7%)
EQTN	Simple Equations and Operations with Literal Symbols	35 (41.7%)	80 (83.3%)
EXPS	Exponents and Square Roots; Scientific Notation	27 (21.1%)	62 (64.6%)
FRAC	Fractions and their Applications	3 (3.6%)	43 (44.8%)
GMMS	Measurement of Geometrical Objects	20 (23.8%)	56 (58.3%)
GRPH	Graphical Representation	13 (15.5%)	59 (61.5%)
INTG	Integers, their Operations & Applications	27 (32.1%)	83 (86.5%)
	Total Number of Students per course	84	96

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IES Panel on Fractions Recommendations

(Siegler et al., 2010)

1. **Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts**
2. **Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward**
3. **Help students understand why procedures for computations with fractions make sense**
4. **Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems**
5. **Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them**

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How to:

- Make the concept relevant
- Make the instruction clear
  - CRA
    - Concrete – hands-on
    - Representational – pictorial of the hands-on
    - Abstract – symbolic notation

    see Witzel & Riccomini's (2008) Computation of Fractions for details
  - Simple to follow algorithms

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Reasoning and Fractions

- Pies
- Pizza
- Ice cube trays
- Fraction strips
  - Fold
  - Shade
  - Define denominator after folding using student language
  - Define numerator after shading using student language
  - How can a teacher show equivalent fractions?

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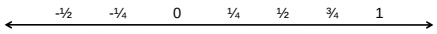
Fractions and Number Paths

Transition to number lines or paths (Siegler, et al., 2010)

- Identify 1/2
- Identify 3/4
- Identify 1 1/3
- Identify -1/5



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Fractions: Concepts and Procedures

- Students must learn computation and procedures.
- Which of these statements do we hear more often when discussing interventions?
  - He doesn't quite understand fractions but he gets it right on the test every time.
  - He seems to understand fractions but he can't get them right on the test.

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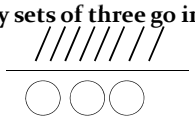
Division with fractional answers

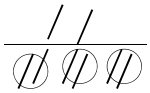
(adapted from Witzel, 2004)

Say, "How many sets of three go into 8?"

8 sticks / 3 cups



Distribute sticks into cups evenly and ask, **"Are there an equal number of sticks per cup? How many sticks per cup?"**



2 sticks per cup with 2 more sticks that need to be divided into 3 cups.

2 sticks / cup + 2 more sticks / 3 cups

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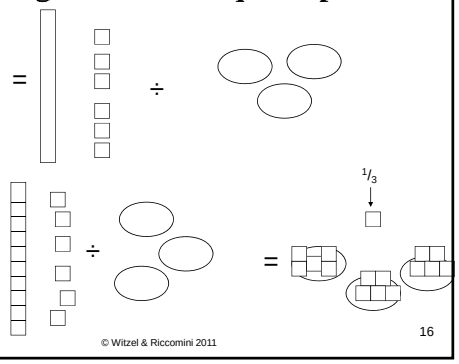
Expanding on the same principles

16 ÷ 3



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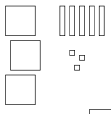
When numbers get bigger, the process does not

355 ÷ 3 =



```
      118  1/3
3 ) 355
    300
     50
     30
     25
     24
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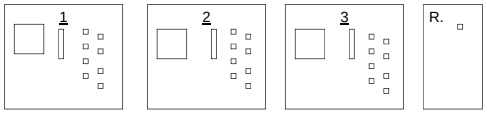

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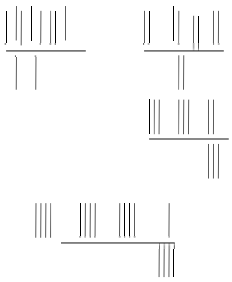
Mixed to improper and back: concrete

- 8 ÷ 2
- 8 ÷ 3
- 13 ÷ 4



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Mixed to Improper Fractions: abstract

$4\ ^{2}/_{5}$ This mixed fraction as an improper fraction is $^{22}/_{5}$.

Why?

Tricks are not helpful to students with memory problems, think about the concept and purpose to the calculations

Say, "Four and two – fifths" "And says to add"

$^{4}/_{1} + {}^{2}/_{5}$ or $^{20}/_{5} + {}^{2}/_{5} = {}^{22}/_{5}$.

This is why you multiply the fraction's denominator and then add the numerator.

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"Simplifying" or reducing fractions

- Adapted from Multisensory Algebra © materials from GreatLeaps / Diarmuid
- These materials can be used to compute addition, subtraction, multiplication, division, negative numbers, fractions, and algebraic expressions and equations
- Assess through manipulations and explanations

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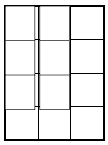
Multiplication of fractions

- Multisensory Algebra
  - Fractional groups of another fraction
- Graph paper
  - Fractions similar to fraction strips
  - Strengths
    - Follows fraction strips
    - A visual representation aids memory
  - Weaknesses
    - Inconsistent process with whole numbers



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Division of Fractions

- Why is it that when you divide fractions, the answer is larger? Also, why do you invert and multiply?

- $^{2}/_{3}$ divided by $^{1}/_{4} = {}^{2}/_{3}\ (^{4}/_{1}) = {}^{8}/_{3}$

$$\frac{^{2}/_{3}\ (^{4}/_{1})}{^{1}/_{4}\ (^{4}/_{1})} = \frac{^{8}/_{3}}{^{4}/_{4}} = \frac{^{8}/_{3}}{^{1}/_{1}} = \frac{^{8}/_{3}}{}$$

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Adding and Subtracting Like and Unlike denominators

- Fraction strips
  - Like denominators

- Multisensory Algebra
  - Like denominators
  - Unlike denominators
    - Requires knowledge of addition, subtraction, multiplication, and division

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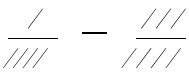
The progression of fractions

- After teaching her students subtraction and negative integers through the fraction computation using sticks, Mrs. Straube was able to transition into more abstract terms

1. 

2. $6\ ^{3}/_{8}$ 6 and $^{3}/_{8}$

   $-\ 2\ ^{7}/_{8}$ $-\ 2$ and $-\ ^{7}/_{8}$

   4 and $-\ ^{4}/_{8}$ $3\ ^{8}/_{8} - {}^{4}/_{8} = 3\ ^{4}/_{8}$

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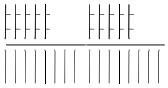
Concrete might get out-of-control

- $^5/_8 \div {}^2/_3$

Divide the numerator into 3 groups per tally mark. Then, count the groups of 2.

$^{15}/_{16}$



Lay down an equivalent fraction until you have a whole number of groups

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Match instruction to how our students think

- When students struggle with multiplication, they likely struggle with long division

```
      1
      4
13 | 75
     52
     23
     13
     10  "Ten that cannot be divided into 13"
```

In this example, the student stacks the numerical answer until calculations work out correctly. The answer is 5 with a remainder of 10 or 10/13.

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Transition

- Proportions

  Ex. Have students compare a picture of themselves to their actual height. The proportion reveals the scale.

  $$\frac{\text{head}}{\text{height}} = \frac{\text{pictured head}}{\text{pictured height}}$$

- Rate (use graph to show)

  $m = \Delta y / \Delta x$

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Resources

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