# Instructional and Error Pattern Considerations

# Part IV

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# Overview Part IV:

- Overview of Error Patterns
- Error Analysis
  - Procedures
- Specific Error Patterns
  - Case Study-Error Patterns
    - Fractions
    - Algebra
- Algebra and Beyond
  - Next Steps

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# Error Analysis

**Error analysis**
- Involves reviewing the student's independent work (e.g., seatwork, homework, CBM probes) to identify specific error types and patterns
- Helps to set priorities for teaching

**Error patterns**
- Indicate areas in need of further instruction
- Constitute a database for determining what content and strategies to teach

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# Error Analysis Process

1. **Identify errors on student's math work**
2. **Categorize errors by type** (for **example**)
   - **Conceptual**
   - **Factual**
   - **Procedural**
   - **Careless**
3. **Look for error patterns within each error type and across each error type**

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# Multiplication Problem

**7 Separate Steps:**

1. **Multiplication Facts:** Are the one-digit multiplication processes completed accurately?
2. **Multiplying All Combinations:** Are all different kinds of multiplication attempted?
3. **Carry (Inside):** Are carries assigned to the proper column?
4. **Carry (Outside):** Is the last carry part of the product?
5. **Adding the Carry:** Are carried numbers combined with the proper column?
6. **Lining up Addition:** Are the intermediate products lined up correctly?
7. **Addition:** Is the final addition process carried out properly.

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# Division Problem

**7 Separate Steps:**

1. **Correct Multipliers**: Are the correct multipliers being chosen?
2. **Multiplication:** Regardless of what multipliers are being chosen, is the multiplication being carried out correctly.
3. **Alignment:** Is the result of the multiplication aligned in the correct position?
4. **Subtraction:** Is subtraction completed correctly?
5. **Bringing Down**: Does the student bring down the correct digit at the appropriate times?
6. **Stopping: Is the procedure stopped at the appropriate time?**
7. **Remained/Fractions**: Are remainders dealt with appropriately?

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## Breakout Activity

Identify the errors for the grade level of one of the below students that you work with:

1. Elementary Grades:
   - John
2. Middle School Grades
   - Ann
3. High School Grades
   - Juan

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## Interview data / Self-analysis

- When students are performing steps to solving an equation, have them perform think-alouds with you.
- Either write down their reasoning behind each step, or
- Have them write down their reasoning in a journal.
- Perform the interview before and after you implement the new form of instruction to determine if significant improvement is made.

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## Error Pattern Analyses

- As math problems become more complex, students need to go through a series of steps to solve problems.
- Often an error in any of these steps can cause failure in the final response.
- As a result, it is our responsibility to determine what the error is and whether the error is conceptual, procedural, or memory related.

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## Summary of Error Analysis

**Error analysis**
- Involves reviewing the student's independent work (e.g., seatwork, homework, CBM probes) to identify specific error types and patterns
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**Error patterns**
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## Algebraic Equations & Beyond: Next Steps

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## Significant Growth towards Algebra

Examined 6th and 7th grade preparedness towards Algebra according the Algebra Readiness Test

Study is limited (38 students with learning disabilities in mathematics; 2 schools in SC)

| Alg Prep | Data/Prob | Equat | Decim | Expon | Fract | Comp | Graph | Integ |
|----------|-----------|-------|-------|-------|-------|------|-------|-------|
| 6th | no | no | no | minimal (ns) | no | minimal (ns) | no | no |
| 7th | Significant Growth | no | no | minimal (ns) | minimal (ns) | no | no | no |

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## Algebra expressions and equations

- Much of the confusion of algebra can be averted through a strong math background.
- However, the abstractness of variables and unknowns confuses many students.
- Hands-on instruction should be completed for concept understanding (e.g. graphical calculators) and task development (e.g. Multisensory Algebra)

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## Multisensory Algebra instruction

- CRA sequence allows hands-on and pictorial exploration of content
- Reinforces arithmetic while covering algebra
- Enforces the concepts within algebra while making the solution appear more available
- Instruction includes researched pedagogical steps as well as an advanced math model

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## Conceptual Framework
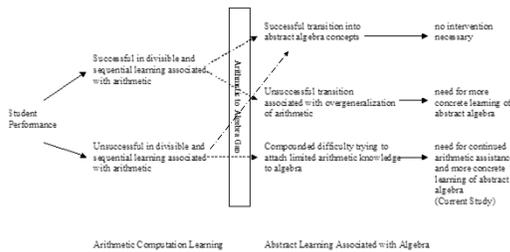(Witzel, Smith, & Brownell, 2001)



Figure 1. Flowchart of Algebraic Needs for Students Who Experience Difficulty in Math

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## Implement CRA instruction in your classroom. Here's how:

- **C**hoose the math topic to be taught
- **R**eview abstract steps to solve the problem
- **A**djust the steps to eliminate notation or calculation tricks
- **M**atch the abstract steps with an appropriate concrete manipulative
- **A**rrange concrete and representational lessons
- **T**each each concrete, representational, and abstract lesson to student mastery (accuracy without hesitation)
- **H**elp students generalize learning through word problems and problem solving events

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## Choose a math topic

- Plan what is to be taught ahead of time.
- Group lessons according to the big idea as determined by your state standards
- Sequence the lessons so they start basic and gradually introduce new topics
- Any math topic can be examined for CRA linkage

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## **R**eview the abstract steps used to solve the problem

- What is the desired math outcome of the group of lessons?
- Determine the procedural goal of the combination of math skills
- List out the steps or procedures
- Remember, not all math skills require abstract knowledge.

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### Adjust the steps to eliminate notation or calculation tricks

- Change or modify steps to create the most logical and sequential set of procedures
- Take a child's point of view when reviewing steps

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### Match the abstract steps with an appropriate concrete manipulative

- Initial understanding of content will be based on interactions with concrete objects, so be careful which ones you choose.
- The conceptual effectiveness of the manipulative object should be noted in accordance to the math skill being taught.
- Avoid concrete objects that only cover a few skills.
- You may have to teach two stages of concrete knowledge
- Also, not all concrete objects are appropriate for CRA instruction. Some materials are effective for conceptual growth while others are useful for procedural work.

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### Arrange concrete and representational lessons

- Practice concrete manipulations. The same questions that you encounter you can be certain your students will as well.
- Practice how to mark pictorial representations that appear similar to concrete manipulations.
- Make certain that your language throughout instruction matches the language required for the desired outcome.

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### Teach each CRA lesson to mastery

- Model and guide students in their use of manipulative objects and pictorial representations.
- Teach students step by step gradually introducing mathematical vocabulary. Allow students to name or invent their stepwise procedures within instruction.
- Move from concrete to representational to abstract learning levels only after students show accuracy without hesitations in manipulations or drawings.
- Assess each level of learning according to stepwise procedures. Take account of students who created different procedures.

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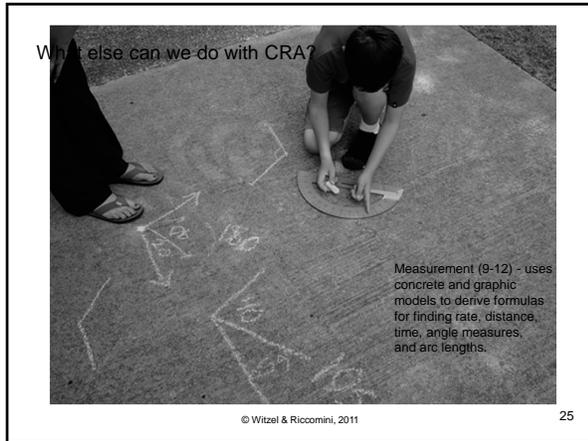### Help students generalize what they learn through word problems

- Incorporate word problems throughout a lesson to help show social relevance as to why a math skill is important to learn
- Use language experiences through the learning process to help prepare for word problem and problem solving application

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### Multisensory Algebra

- Reducing Expressions

- Inverse Operations

- Transformational Equations

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What else can we do with CRA?

Measurement (9-12) - uses concrete and graphic models to derive formulas for finding rate, distance, time, angle measures, and arc lengths.

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## Your turn with Algebra

- *Students solve linear equations and inequalities.*
  - MA.912.A.3.2 Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

- *Students write equations and draw graphs of conic sections (circle, ellipse, parabola, and hyperbola), thus relating an algebraic representation to a geometric one.*
  - MA.912.A.9.1 Write the equations of conic sections in standard form and general form, in order to identify the conic section and to find its geometric properties (foci, asymptotes, eccentricity, etc.).

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## Arguments for and against algorithms

- Algorithmic instruction receives both admonishment and celebration, often by the same researcher.
- The most current argument against algorithms have been that they lead to blind adherence to stepwise rules without thought.

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## Improving on algorithms

- Teach the process of the algorithm
- Allow students to interact with the procedures
- Oversee that the algorithm can cover future work that may appear similar to the current skill.
- Make the algorithm easy to remember
  - ex. PEMDAS; ROYGBIV

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## Reduce Meaningless Algorithms

- Students with memory concerns often receive remediation in the form of memory-based learning.
- For example, a child who can't learn the multiplication tables starts remediation with timed quizzes and flashcards. While this may be motivational, it is not necessarily instructional.

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## Clues and Undo's for procedures

$\frac{5}{6}X + 4 = 8,$
solve for X

|  | Clues | Undos |
|---|---|---|
|  | Multiply by $\frac{5}{6}$ | Divide by $\frac{5}{6}$ |
|  | Add 4 | Subtract 4 |

Algebraic - The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions. (MA.D.1.3)
Operations - selects the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers, ratios, proportions, and percents, including the appropriate application of the algebraic order of operations. (MA.A.3.3)

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## Quadratic Equation example

$3x^2 + 10x - 8$

1. $(A)(C) = (3)(^-8) = ^-24$
2. Factors of AC

   $^-6$ & $4$; $^-4$ & $6$; $^-8$ & $3$; $^-3$ & $8$; $^-12$ & $2$; $^-2$ & $12$
3. Sum or Difference to B

   $^-2 + 12 = 10$
4. Substitute for B

   $3x^2 - 2x + 12x - 8$
5. Solve by groups

   $x(3x-2) + 4(3x-2)$

   $(x+4)(3x-2)$

1. AC

   $(^-8)(3) = ^-24$
2. Factors that sum to B

   $^-8 + 3 = -5$ ---NO

   $+12 + ^-2 = +10$ --- YES
2. Divide the factors by A

   $12/3 = 4$ and $^-2/3$
3. Solve for X; $x=^-4$ and $x=2/3$
4. Rewrite factors

   $(x+4)(3x-2)$

Mike Diamond,
SC math teacher

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## Let's Make Some Mnemonics and Acrostics

### Quadratic
- Find out if the formula is quadratic
- Accurately compute to equal the whole number
- Compute the same numbers to equal the coefficient
- The parentheses come next
- Order what's in the parentheses
- Recheck the answer

### Slope-Intercept
- Recognize and reorder the formula
- Identify the slope and intercept
- Set up the intercept
- Evaluate the slope
- Rise over run
- Use the dots to make a line
- Need to check your answer

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## Sample strategies

**I**dentify the variables
**S**et up equations
**O**rganize to balance
**L**et equations begin
**A**dd variable side of equal sign
**T**otal other side
**E**valuate and check answer

**F**ind what you are solving
**A**sk yourself about the parts
**S**et up the numbers
**T**ie down the sign

**D**iscover the sign
**R**ead the problem
**A**nswer or draw or check
**W**rite the answer

What algorithms can you turn into learning strategies?

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## Error Patterns through teacher observation

Go beyond right and wrong and into the why behind student answers

1. Task Analysis: Create a checklist of steps required to solve a problem
2. Observe student work according to the checklist of steps
   - When grading student work
   - When watching or interviewing a student working
3. Reteach according to any patterns of incorrect work

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## Task Analyzing your Curriculum

1. Predict the optimum sequence to reach the outcome your textbook's chapter before you begin teaching
2. Match your task analysis to the textbook
3. Note commonalities and differences
4. Check earlier chapters to see if they cover the differences. Check later chapters to see if they cover the differences.
5. Check supplemental guides to see if they cover the differences
6. Develop additional instruction to complement the current text / curriculum
7. Sequence the instruction as your students need

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