The Importance of Number Sense and Overall Math Proficiency

2011 Numeracy Conference

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Agenda

- Introduction
- Number Sense Defined
  - Counting forward and backwards
  - Fluent quantification and magnitude of number
  - Number to numeral identification
  - Base-10 and place recognition and recall
  - Fluent use of arithmetic strategieS
- Instructional Recommendations

U.S. Math Performance

The 2005 & 2007 National Assessment of Educational Progress (NAEP) reported:

- 15% of Grade 4 students scored below the basic level
- 25% of Grade 8 students scored below the basic level
- 36% of Grade 12 students scored below the basic level

U.S. Math Performance

The 2005 & 2007 National Assessment of Educational Progress (NAEP) reported:

- 40% of Grade 4 students with disabilities scored below the basic level
- 66% of Grade 8 students with disabilities scored below the basic level
- 83% of Grade 12 students with disabilities scored below the basic level

Curricular Content

Streamline the Mathematics Curriculum in Grades PreK-8:

- Follow a ***Coherent Progression***, with Emphasis on ***Mastery of Key*** Topics
- Focus on the Critical Foundations for Algebra
  - ***Proficiency with Whole Numbers***
  - ***Proficiency with Fractions***
  - Particular Aspects of ***Geometry and Measurement***
- Avoid Any Approach that Continually Revisits Topics without Closure (pg 22)

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Where the struggle begins

Geary, Hoard, and Hamson (1999) found that first-grade students with disabilities in mathematics already show difficulties with counting knowledge, number naming and writing and memory retrieval as compared to their nondisabled peers.

Geary and his colleagues argued that these early errors in basic math may affect their future mathematics learning.

Gersten and Chard (1999) labeled the problems listed by Geary and his colleagues as difficulty with **number sense**.

Number Sense Defined

Number sense is an emerging construct (Berch, 1998) that refers to a child's fluidity and flexibility with numbers, the sense of what numbers mean and an ability to perform mental mathematics and to look at the world and make comparisons."

– - Russell Gersten & David Chard

Number Sense defined

In its most fundamental form, number sense entails an ability to immediately identify the numerical value associated with small quantities;

- …this more highly developed form of number sense should extend to numbers written in fraction, decimal, and exponential forms.
- …poor number sense interferes with learning algorithms and number facts and prevents use of strategies to verify if solutions to problems are reasonable.
- NMAP, page 27, March 2008

Examples of Number Sense

Do you have a sense of number?

- Is 4 x 12 closer to 40 or 50?
- How many paper clips can you hold in your hand?
- If the restaurant bill was $119.23, how much of a tip should you leave?
- How long will it take to make the 50 mile drive to Washington, D.C.?
- If a 10-year old is 5' tall, how tall will the child be at age 20?

Fennell, 2008

Mathematics Performance

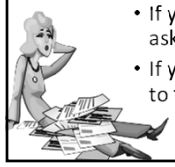
Translated to Real World Performance

- 78% of adults cannot explain how to compute interest paid on a loan
- 71% cannot calculate miles per gallon
- 58% cannot calculate a 10% tip
- 27% of 8th graders could not correctly shade 1/3 of a rectangle
- 45% could not solve a word problem that required dividing fractions

Mathematics Advisory Panel Final Report, 2008

Why focus on BIG IDEAS in Number Sense?

- Intensive instruction means teach less more thoroughly
  - If you don't know what is important, everything is.
  - If everything is important, you will try to do everything.
  - If you try to do everything you will be asked to do more.
  - If you do everything you won't have time to figure out what is important.



Areas within Number Sense

(Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005; Fuson, 1990, Gersten, Jordan, & Flojo, 2005)

a) counting forward and backwards
   - Strategic Counting

b) fluent quantification and magnitude of number

c) number to numeral identification

d) base-10 and place recognition and recall

e) fluent use of arithmetic strategies
   a) Retrieval of basic arithmetic facts

Focus on these in your assessment of early learners

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Manipulative Objects: A Number Sense Teaching Tool

- Researched benefits span over:
  - Developing numeration -Basic facts -Fractions
  - negative #s -Area & perimeter -3D figures
- Manipulative objects do NOT teach children…. Teachers do!
- Some helpful hints
  - Practice using manipulatives before you teach
  - Provide language experiences while using manipulatives
  - Develop a pictorial representation for transition to abstract understanding… the ultimate purpose

Make Numbers Relevant

- Cooking
- Hop scotch
- Calendar
- Sport scores
- Game boards
- Color by number

Some Basic Skills

a) Counting and counting on
- Counting chart, unifix cubes, feed the monkey

a) Counting backwards (difficult until early elementary)
- Counting chart, unifix cubes

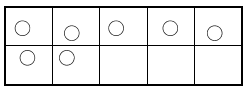
b) Ten frames to aid number sense

- Difficulty grouping numbers
- Building numbers and numerals
- Transition to operations

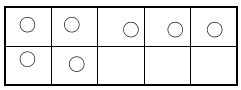
Ten Frames

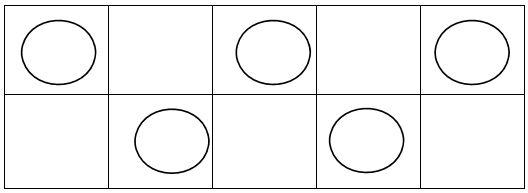
b) Using patterns to earn numeracy skills and numbers

- 3+4=7



- 5+2=7





3 + 2 = 5
Can you represent another way?
2 + 3 =5
Another way?
4 + 1 = 5

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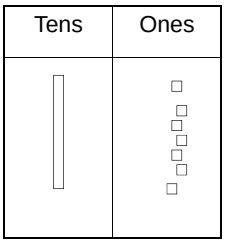
Language Experiences

d) Base ten



Using Spatial Sense to teach Vocab

- In, over, under, next to, beside, out, right and left
- Boy in a box game
- Generalize the language / concept to use with a figurine and a cube

A Checklist of a Numerically Powerful Child:

1. Develops MEANING for numbers and operations.

a. Connects numerals with situations from life experiences.
b. Knows that numbers have multiple interpretations.
c. Understands that numbers size is relative.
d. Connects addition, subtraction, multiplication, and division with actions arising in real-world situations.
e. Understands the effects of operating on numbers.
f. Creates appropriate representations for numbers.
g. Creates appropriate representations for operations.

A Checklist of a Numerically Powerful Child:

2. Looks for RELATIONSHIPS among numbers and operations.

a. Decomposes or breaks apart numbers in different ways.
b. Knows how numbers are related to other numbers.
c. Understands how the operations are connected to each other.
   - Multiplication/Division – Inverse
   - Addition/Subtraction – Inverse
   - Multiplication - Repeated Addition
   - Division - Repeated Subtraction.

A Checklist of a Numerically Powerful Child:

3. UNDERSTANDS computation strategies and uses them appropriately and efficiently.

a. Correctly performs the steps in an algorithm and can discuss why the algorithm works.
b. Makes a conscious effort to complete calculations using prior knowledge and simpler calculations.
c. Often uses a variety of calculation strategies, even when completing calculations involving the same operation.
d. Chooses appropriate calculation technique to obtain exact answers and estimate.
e. Calculates with accuracy and relative efficiency.

A Checklist of a Numerically Powerful Child:

4. Makes sense of numerical and quantitative situations.

a. Expects numerical calculations to make sense.
b. Seeks to understand relationships among quantities in real-world situation.
c. Assesses whether the result of a calculation makes sense in the context of the numbers and real-world quantities involved.

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Breakout Activity

- Working with a colleague, review each of the 5 areas for Numerically powerful children and discuss the greatest weaknesses your students display in each area.
- Discuss ways in which you or your school's math program addresses these areas.

Learning Processes

- To prepare students for Algebra, the curriculum must ***simultaneously*** develop ***conceptual understanding***, ***computational fluency***, ***factual knowledge*** and ***problem solving skills***.
- Limitations in the ability to keep many things in mind (***working-memory***) can hinder mathematics performance.
  - ***Practice*** can offset this through automatic recall, which results in less information to keep in mind and frees attention for new aspects of material at hand.
  - Learning is most effective when ***practice is combined with instruction*** on related concepts.
  - Conceptual understanding ***promotes transfer*** of learning to new problems and better long-term retention.

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Why learn Facts?

1. Knowledge of simple facts is needed for proper use of calculators
2. Ability to estimate implies mastery of single digit facts
3. Students slow at facts are less likely to learn more complex math problem types
4. Students must know multiplication facts quickly to be able to master fractions
5. Algebra is not open to those who haven't mastered fractions.

Crawford, 2002

Basic Fact Activities

- **Instructional Activities to Teach Basic Facts**
  - **Activities for *Understanding***
  - **Activities for *Relationship building***
  - **Activities for *Mastery***
    - ***fluency and automaticity***

Basic Facts

Activities for Mastery (Fluency)

Requires

1. Specific criterion for introducing new facts
2. Intensive practice on newly introduced facts (more than 1x)
3. Systematic practice on previously introduced facts
4. Adequate allotted time (10min/day)
5. Record keeping
6. Motivational procedures

Note: While pictures (including number lines) aid students in using a counting strategy, PICTURES ARE A DETERRENT TO MEMORIZATION and should not be permitted during fluency instruction

Fact Fluency Practice

- Peer-mediated activities
  - Are activities that include a set of instructional procedures where by students are taught by peers
  - Students work together through a series of structured activities to practice important skills during peer-mediated instructional time

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Just an Example

- Example sheet from Otter Creek Institute's Mastering Math Facts Program
- Students use the top to practice
  - Peer-mediated
  - Teacher directed
  - Individual practice
- Bottom half is their timed test
- Goal set in advance
- Performance is graphed
- Student progress on individual pace

Summary on Number Sense

- It is critical in the elementary levels for students to develop proficiency in each of these areas:
  a) counting forward and backwards
     - Strategic Counting
  b) fluent quantification and magnitude of number
  c) number to numeral identification
  d) base-10 and place recognition and recall
  e) fluent use of arithmetic strategies
     - Retrieval of basic arithmetic facts
- Number sense can be taught
- Increase Opportunities to engage in mathematics in a variety of well structured and explicit activities
- Confirm teachers have an understanding of what number sense is and the belief that it can be taught.

Summation

- Students very often feel that math is boring, too difficult, or does not connect to real-life.
- Our charge is to prepare children for success and enjoyment of mathematics. Effective lessons build success. Success builds confidence. Our children deserve at least that.
- It is our job to prepare students to be successful in math at the next level. Build understanding to prepare for the next grade, next task, or next concept.

Early Childhood Math Development

Useful Resources

- http://www.state.tn.us/education/ci/cistandards2001/earlychildhood/sec3math.pdf
- http://www.myscschools.com/offices/cso/standards/math/default.cfm
- http://naeyc.org/about/positions/pdf/psmath.pdf

Number Books

- http://www.mathstories.com/
- http://www.harcourtschool.com/menus/auto/13/1.html

Some e-help

http://www.pilotmath.com

More websites (adapted from Ameis, 2006)**:**

- http://www.canteach.ca/elementary/math.html
  quick reference for lesson ideas
- http://mathforum.org
  question and answer sessions through resources such as Dr. Math
- www.nationalmathtrail.org
  Tutorials and lesson plans are available
- www.coolmath.com
  online questions for students to apply problem solving strategies
- http://www.teachers.ash.au.or.au/jeather/maths/dictionary.html
  online and interactive math dictionary
- www.playkidsgames.com/mathGames.htm
  Games for math facts and early level learning
- http://www.learner.org/teacherslab/math/patterns/index.html
  Early algebraic patterns for students to explore
- www.eduplace.com/math/brain/
  Brain teasers in math
- http://nlvm.usu.edu/en/nav/vlibrary.html
  Virtual Math Library

References

Ball, C. (2003). Every stage counts in the journey to numeracy. *Times Education Supplement* (Oct. 10, 2003), 24.

Gersten, R., & Chard, D. (1999). Number sense: rethinking arithmetic instruction for students with mathematical disabilities. *Journal of Special Education, 33 (1)*, 18-28.

Geary, D. C., Hoard, M. K., & Hamson, C. O. (1999). Numerical and arithmetical cognition: patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology, 74 (3),* 213-239.

Huber, L. L., & Lenhoff, R. S. (2006). Mathematical concepts come alive in pre-k and kindergarten classrooms. *Teaching Children Mathematics, 13*, 226-231.

Ferguson, C. (2001). Discovering, supporting, and promoting young children's passions and interests: One teacher's reflections. *Young Children, 56*, 6-11.

Kamii, C. (2000). *Young children reinvent arithmetic* (2nd ed.). New York: Teachers College Press.

National Council of Teachers of Mathematics [NCTM], (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

Smith, S. S. (2006). *Early childhood mathematics* (3rd ed.). Boston: Allyn & Bacon.

Witzel, B. S., & Ferguson, C. J. (in-review). Developing early number sense for students with disabilities.

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