Research Supported Strategies for Instruction and Intervention: Number Sense through Algebra

Numeracy Workshop for Teachers

March 3, 2011

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Overview Part I:

- Introductions
- National Mathematics Advisory Panel Report
  - US Math Performance
  - Curricular Content
  - Learning Processes
  - Instructional Practices
- Learner Characteristics
  - Strategic Learner vs. Non-Strategic Learner
- Number Sense
  - Underlying Foundation
    - Quantity Comparison and Counting
    - Computation

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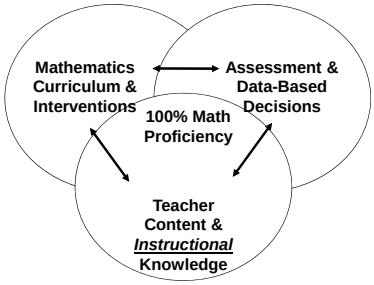
Framework for Effective Mathematics Instructional Programs



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Foundations for Success

National Mathematics Advisory Panel

Final Report, March 2008

Select Slides taken from the NMAP-Final Report
Presentation available at: **http://www.ed.gov/MathPanel**

U.S. Math Performance

The 2005 & 2007 National Assessment of Educational Progress (NAEP) reported:

- 15% of Grade 4 students scored below the basic level
- 25% of Grade 8 students scored below the basic level
- 36% of Grade 12 students scored below the basic level

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U.S. Math Performance

The 2005 & 2007 National Assessment of Educational Progress (NAEP) reported:

- 40% of Grade 4 students with disabilities scored below the basic level
- 66% of Grade 8 students with disabilities scored below the basic level
- 83% of Grade 12 students with disabilities scored below the basic level

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Math Proficiency of U.S. Students

- International comparisons
- Low fractions of proficiency on NAEP
- Falling proficiency at higher grades
- Heavy remedial demand upon entry into college
- Achievement gap

Algebra as a gateway

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Mathematics Performance

Translated to Real World Performance

- 78% of adults cannot explain how to compute interest paid on a loan
- 71% cannot calculate miles per gallon
- 58% cannot calculate a 10% tip
- 27% of 8th graders could not correctly shade 1/3 of a rectangle
- 45% could not solve a word problem that required dividing fractions

Mathematics Advisory Panel Final Report, 2008

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Curricular Content

Streamline the Mathematics Curriculum in Grades PreK-8:

- Follow a ***Coherent Progression***, with Emphasis on ***Mastery of Key*** Topics
- Focus on the Critical Foundations for Algebra
  - ***Proficiency with Whole Numbers***
  - ***Proficiency with Fractions***
  - Particular Aspects of ***Geometry and Measurement***
- Avoid Any Approach that Continually Revisits Topics without Closure (pg 22)

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Learning Processes

- To prepare students for Algebra, the curriculum must ***simultaneously*** develop ***conceptual understanding***, ***computational fluency***, ***factual knowledge*** and ***problem solving skills***.
- Limitations in the ability to keep many things in mind (***working-memory***) can hinder mathematics performance.
  - ***Practice*** can offset this through automatic recall, which results in less information to keep in mind and frees attention for new aspects of material at hand.
  - Learning is most effective when ***practice is combined with instruction*** on related concepts.
  - Conceptual understanding ***promotes transfer*** of learning to new problems and better long-term retention.

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Instructional Practices

Instructional practice *should be informed by high quality research,* when available, and by the best professional judgment and experience of accomplished classroom teachers.

- All-encompassing recommendations that instruction should be *student-centered* or *teacher-directed* ***are not supported by research.***

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Instructional Practices

Research on students who are *low achievers, have difficulties in mathematics, or have learning disabilities* related to mathematics tells us that the effective practice includes:

- ***Explicit methods of instruction available on a regular basis***
- ***Clear problem solving models***
- ***Carefully orchestrated examples/ sequences of examples.***
- ***Concrete objects to understand abstract representations and notation.***
- ***Participatory thinking aloud by students and teachers.***

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For More Information

Please visit us online at:
http://www.ed.gov/MathPanel

- **Read it! Fact Sheet Supplemental Handout #1**
- **The report and Factsheet should be on the desk of every teacher responsible for teaching and planning math.**
- **It is essential that teachers instructional decisions are guided by the best available evidence, not only by philosophy**

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Learner Characteristics

- Strategic Learners
  - Able to analyze a problem and develop a plan
  - Able to organize multiple goals and switch flexibly from simple to more complicated goals
  - Access their background knowledge and apply it to novel tasks
  - Develop new organizational or procedural strategies as the task becomes more complex
  - Use effective self-regulated strategies while completing a task
  - Attribute high grades to their hard work and good study habits
  - Review the task-oriented-goals and determine whether they have been met

http://iris.peabody.vanderbilt.edu/srs/chalcycle.htm

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Learner Characteristics

- Non-Strategic Learners
  - Unorganized, impulsive, unaware of where to begin an assignment
  - Unaware of possible steps to break the problem into a manageable task, possibly due to the magnitude of the task
  - Exhibit problems with memory
  - Unable to focus on a task
  - Lack persistence
  - Experience feelings of frustration, failure, or anxiety
  - Attribute failure to uncontrollable factors (e.g., luck, teacher's instructional style)

http://iris.peabody.vanderbilt.edu/srs/chalcycle.htm

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Essential Question for Teachers

- Essential Question
  - What did I do differently for the struggling students?
- Asked during instructional planning and after instructional delivery!!!

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Early Numeracy Skills for Students with Disabilities

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The number code

Some people say that they like math because it logical....

Find the pattern in each of these groups of numbers.
Is the pattern visual, auditory, both, or neither?

A. 1-9

B. 10-19

C. 20-29

D. 100-120

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Who is at fault?

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The college professor said, "Such wrong in the student is a shame,
Lack of preparation in high school is to blame."
Said the high school teacher, "Good heavens, that boy is a fool.
The fault, of course, is with the middle school."
The middle school teacher said, "From such stupidity may I be spared, They send him to me so unprepared."
The elementary teacher said, "The kindergartners are block-heads all. They call it preparation; why, it's worse than none at all."
The kindergarten teacher said, "Such lack of training never did I see, What kind of mother must that woman be."
The mother said, "Poor helpless child, he's not to blame
For you see, his father's folks are all the same."
Said the father, at the end of the line,
"I doubt the rascal's even mine!"

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Where the struggle begins

Geary, Hoard, and Hamson (1999) found that first-grade students with disabilities in mathematics already show difficulties with counting knowledge, number naming and writing and memory retrieval as compared to their nondisabled peers. Geary and his colleagues argued that these early errors in basic math may affect their future mathematics learning.

Gersten and Chard (1999) labeled the problems listed by Geary and his colleagues as difficulty with **number sense**.

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Number Sense defined

The NMP defined number sense as "an ability to immediately identify the numerical value associated with small quantities, a facility with basic computing skills, and a proficiency in approximating the magnitudes of small numbers of objects and simple numerical operations" (2008, 27).

More advanced forms of number sense involve understanding of place value, composition and decomposition of number, and the concept of basic arithmetic operations (NMP).

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Areas within Number Sense

(Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005; Fuson, 1990, Gersten, Jordan, & Flojo, 2005)

a) counting forward and backwards
b) fluent quantification and magnitude of number
c) Numeral and number identification
d) base-10 and place recognition and recall
e) fluent use of arithmetic strategies

Focus on these in your assessment of early learners

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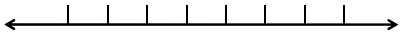
Some Basic Skills

a) Counting and counting on
- Counting chart, unifix cubes, feed the monkey

a) Counting backwards (difficult until early elementary)
- Counting chart, unifix cubes



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b) Quantification and Magnitude

- Building numbers and numerals
  - Visual representations
- Prepare Transition to operations
  - From counting to operational fluency

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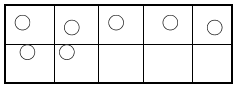
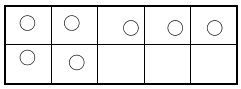
Ten Frames

b) Using patterns to earn numeracy skills and numbers

- 3+4=7



- 5+2=7



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c) Number to Numeral connection

- Be explicit with the naming
- Be explicit as to the value / magnitude

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Numeral and Number

5 =

3 =

Subitization – a fast and accurate observation of number of items

Possibly even with dot notation

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Be explicit as to the connections

- 1.34 1u + 3tths + 4hths
  +0.69 + 0u+ 6tths + 9hths
  1u + 9tths +13hths

1 +(.90+.13)
1 + 1.03
2.03

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d) Place value and base ten

- From 2 digit place value to scientific notation, place value must be understood so that numbers are better understood.
- Incorporate place value language
- Prepare computation in groups of 10
  - Establish 10 as a base number to develop place value 6+4=10; 10-3=7

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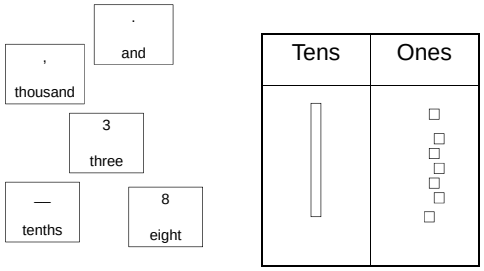
Language Experiences



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e) Fluent Arithmetic Strategies

From counting on fingers and number lines to numeral recognition, the goal is to build computation accuracy, fluency, and automaticity and understand such computation.

- Concrete to Representational to Abstract Sequence of Instruction (CRA) for accuracy
- Repetition and practice for fluency
- Verbal practice for automaticity

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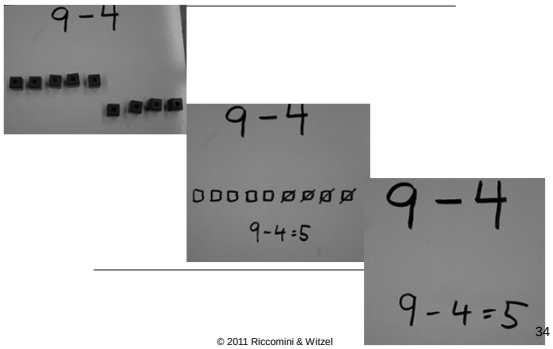
Concrete to Representational to Abstract



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There is more: Using Spatial Sense to teach Vocab

- In, over, under, next to, beside, out, right and left
- Boy in a box game
- Generalize the language / concept to use with a figurine and a cube

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Make Numbers Relevant

- Cooking
- Hop scotch
- Calendar
- Sport scores
- Game boards
- Color by number

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Don't teach harder

- It is our job to prepare students to be successful in math at the next level. Build understanding to prepare for the next grade, next task, or next concept.
- If we can be of any assistance, please do not hesitate to ask.

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