

Supplemental Handouts

Research Supported Strategies for Instruction and Intervention: Number Sense through Algebra

Numeracy Workshop for Teachers
Wichita, KS

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Handout 1

National Math Panel

March 2008

To compete in the 21st century global economy, knowledge of and proficiency in mathematics is critical. Today's high school graduates need to have solid mathematics skills—whether they are headed for college or the workforce. To help ensure our nation's future competitiveness and economic viability, President George W. Bush created the National Mathematics Advisory Panel (National Math Panel) in April 2006.

The panel was charged with providing recommendations to the President and U.S. Secretary of Education Margaret Spellings on the best use of scientifically based research to advance the teaching and learning of mathematics. Expert panelists, including a number of leading mathematicians, cognitive psychologists, and educators, reviewed numerous research studies before preparing a final report containing guidance on how to improve mathematics achievement for all students in the United States.

The National Math Panel's final report, issued on March 13, 2008, contains 45 findings and recommendations on numerous topics including instructional practices, materials, professional development, and assessments. Highlights from the report are briefly summarized below. Please visit www.ed.gov/MathPanel for the executive summary and full report.

Core Principles of Math Instruction

- The areas to be studied in mathematics from pre-kindergarten through eighth grade should be streamlined and a well-defined set of the most important topics should be emphasized in the early grades. Any approach that revisits topics year after year without bringing them to closure should be avoided.
- Proficiency with whole numbers, fractions, and certain aspects of geometry and measurement are the foundations for algebra. Of these, knowledge of fractions is the most important foundational skill not developed among American students.
- Conceptual understanding, computational and procedural fluency, and problem solving skills are equally important and mutually reinforce each other. Debates regarding the relative importance of each of these components of mathematics are misguided.
- Students should develop immediate recall of arithmetic facts to free the "working memory" for solving more complex problems.
- The benchmarks set forth by the Panel should help to guide classroom curricula, mathematics instruction, textbook development, and state assessments.
- More students should be prepared for and offered an authentic algebra course at Grade 8.
- Algebra should be consistently understood in terms of the "Major Topics of School Algebra," as defined by the National Math Panel.
- The Major Topics of School Algebra include Symbols and Expressions; linear equations; quadratic equations; functions; algebra of polynomials; and combinatorics and finite probability.

Student Effort Is Important

Much of the public's "resignation" about mathematics education is based on the erroneous idea that success comes from inherent talent or ability in mathematics, not effort. A focus on the importance of effort in mathematics learning will improve outcomes. If children believe that their efforts to learn make them "smarter," they show greater persistence in mathematics learning.

Importance of Knowledgeable Teachers

- Teachers' mathematical knowledge is important for students' achievement. The preparation of elementary and middle school teachers in mathematics should be strengthened. Teachers cannot be expected to teach what they do not know.
- The use of teachers who have specialized in elementary mathematics teaching could be an alternative to increasing all elementary teachers' mathematics content knowledge by focusing the need for expertise on fewer teachers.

Effective Instruction Matters

- Teachers' regular use of formative assessments can improve student learning in mathematics.
- Instructional practice should be informed by high-quality research, when available, and by the best professional judgment and experience of accomplished classroom teachers.
- The belief that children of particular ages cannot learn certain content because they are "too young" or "not ready" has consistently been shown to be false.
- Explicit instruction for students who struggle with math is effective in increasing student learning. Teachers should understand how to provide clear models for solving a problem type using an array of examples, offer opportunities for extensive practice, encourage students to "think aloud," and give specific feedback.
- Mathematically gifted students should be allowed to accelerate their learning.
- Publishers should produce shorter, more focused and mathematically accurate mathematics textbooks. The excessive length of some U.S. mathematics textbooks is not necessary for high achievement.

Effective Assessment

The National assessment of Educational Progress (NAEP) and state assessments in mathematics should be improved in quality and should emphasize the most critical knowledge and skills leading to Algebra.

Importance of Research

The nation must continue to build the capacity for more rigorous research in mathematics education to inform policy and practice more effectively.

For more information, please visit www.ed.gov/mathpanel.

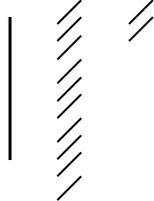
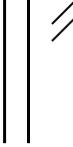
Samples of CRA Manipulative Work:

Adding

$5 + 17$

 + 

To perform this addition students must place toothpicks and sticks next to each other and count-on

=  or 

$2N + 3N$

 + 

To perform this addition of coefficients students must add the number of cups with the variable



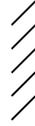
Subtracting

$$8 - 3$$

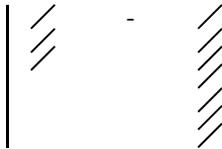


Take away subtraction means you start with the first digit and take away the

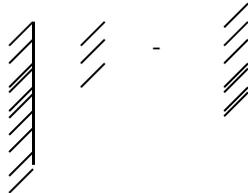
Second number to end with



$$13 - 7$$



Simple subtraction involves grouped subtraction by taking away toothpicks or sticks simultaneously. In this case the student needs to borrow from the tens place first.



After the students substitute 10 toothpicks for one Popsicle stick, then they perform grouped subtraction.



The toothpicks left are not to the right of the negative (minus) sign so the answer must be positive.

$$-4X + 2X$$

$$- \text{○○○○}X + \text{○○}X$$

Coefficients also are subtracted in groups

$$- \text{○○}X + X$$

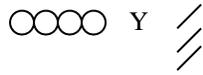
The answer is $-2X$. Students must be explained that the blanks next to the X mean that the coefficients are now 0. The coefficients have been subtracted and thus 0 groups remain; $X = 0$. Remove the X to the right.

$$- \text{○○}X$$

***** If needed, explain that 0 X occurs as a result of the subtraction. A numerical expression such as $X + 3 = 7$ does not mean $X = 0$ unless stated in the text as $0X$. *****

Multiplication

$$4Y(3)$$



This odd grouping pattern means 4 groups of Y multiplied by 3. We do not know what Y is equal to so we will not manipulate that further, but we can manipulate the 3. Students need to fill **each** cup with 3 toothpicks.



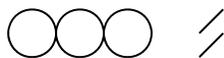
Since multiplication involves totals have the students count total toothpicks. In this case there are 12 toothpicks. Thus the statement changes to 12 groups of Y.



Although long, this expression represents 12 groups of Y.

Other multiplication procedures may be carried similarly using the first number as groups and the second as a number.

$$3(2)$$



becomes



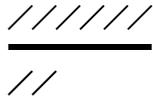
The total number of toothpicks is 6 thus making the final number 6 represented by 6 toothpicks.



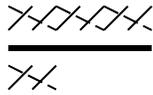
Division

Students must have an understanding of division to obtain correct answers.

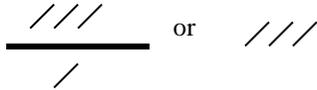
$$\frac{6}{2}$$



Unlike subtraction, where toothpicks are simultaneously pulled from both groups, division requires students to recognize subgroups. In this case the denominator divides into the numerator with no remainder. Students may perform this by recognizing the grouping of 2.

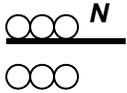


Students may cross lines through the groups of 2 (or simply circle groups) to label that 3 groups exist in the numerator and 1 in the denominator.

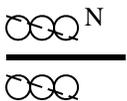


This process works similarly for coefficients

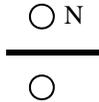
$$\frac{3N}{3}$$



Since the numerator and denominator may be grouped by 3, all the groups of 3 groups may be crossed out or circled.



In this case it is one group of X over one group of X. Therefore, the expression may be represented as 1 / 1 N or simply 1N.

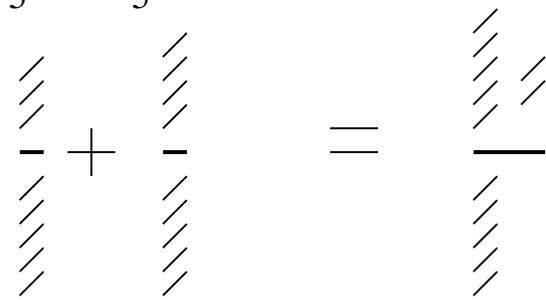


or $\bigcirc N$

Fractions

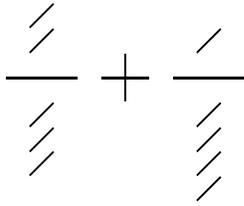
Not all manipulative objects can be used for conceptual development and task learning. This is a concern with many “Reform” math programs.

Adding fractions with like denominators

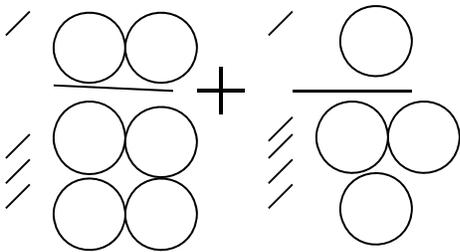
$$\frac{3}{5} + \frac{4}{5}$$


Adding fractions with like denominators (requires multiplication of fractions)

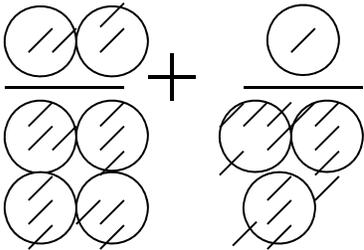
$$\frac{2}{3} + \frac{1}{4}$$



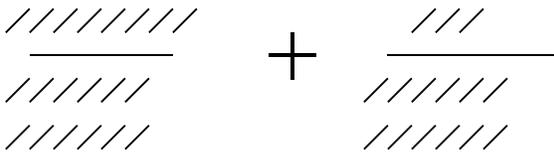
Use tally marks or onesticks to represent the numerators and denominators



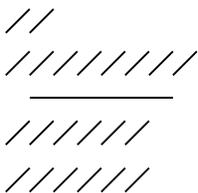
Multiply each fraction by the other's denominator using cups



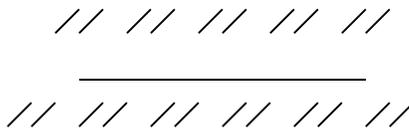
Discuss the definition of multiplying the number of tallies or onesticks within each cup



After counting and removing the cups the numerators are set and the denominators are equal. The fractions are ready to add.



The initial answer is $\frac{10}{12}$



The numerator and denominator are grouped by the common factor of two to reveal 5 groups over 6. Thus, the answer is $\frac{5}{6}$.

Fractions using arrays

$$\left(\frac{1}{3}\right)\left(\frac{2}{4}\right)$$

Algebraic expressions and equations

Reducing Expressions

$$5 - 2X - 6$$

step 1

Set up equation with concrete objects as shown here

step 2

Reorder the equation to place the numbers next to each other.

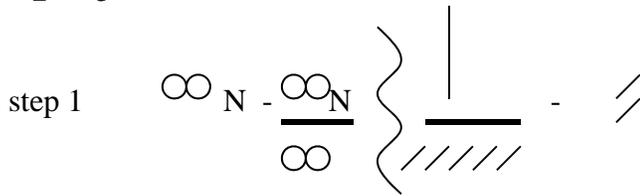
step 3

Complete grouped subtraction of +5 and -6 to determine the answer. -2X is not affected as there are no other X variables.

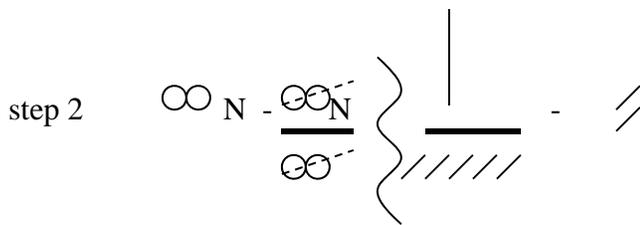
Transition to Inverse Operations

Describe / Model

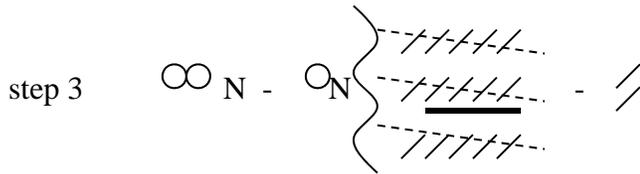
$$a) 2N - \frac{2N}{2} = \frac{10}{5} - 2$$



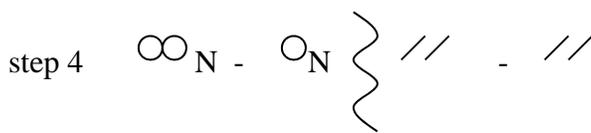
Set up equation with concrete objects as shown here



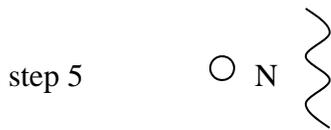
Point out that there are an equal number of numerator and denominator coefficients for N thus making it 1N.



After substituting 10 onesticks for one tenstick, point out that there are two groups of 5 in the numerator and 1 group of 5 in the denominator resulting in 2.



The equation is left with 2N-1N = 2-2



Reduce the left side of the equation using subtraction to make 1N. Reducing the right side of the equation results in 0.

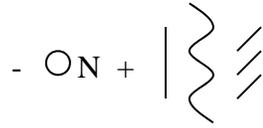
$$N = \bigcirc$$

The result is N = 0 since N is empty

Inverse Operations

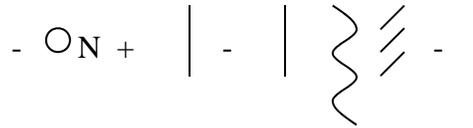
$$-N + 10 = 3$$

step 1



Set up equation with concrete objects as shown here

step 2



Subtract 10 from each side of the equation to isolate the variable N

step 3



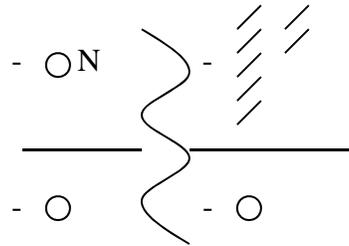
Since $3 - 10$ maybe difficult for some students, borrow ten ones to replace the 10 marker.

step 4



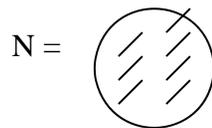
After grouped subtraction the student has $-1N = -7$

step 5



Divide both sides by negative one

step 6



Close-up:
Since a negative divided by a negative is a positive the negatives can be turned over. Also, the seven ones sticks are distributed among one cup. The answer is $N = 7$ sticks.

Determining Error Patterns

1. Task Analysis of Math Skill

Discussion of standards movement has helped but there are more details that a teacher must address to effectively transition from skill to skill.

1. Predict the optimum sequence to reach the outcome your textbook's chapter before you begin teaching
2. Match your task analysis to the textbook
3. Note commonalities and differences
4. Check earlier chapters to see if they cover the differences. Check later chapters to see if they cover the differences.
5. Check supplemental guides to see if they cover the differences
6. Develop additional instruction to complement the current text / curriculum
7. Sequence the instruction as your students need

2. Observe Student Work

Solving for single variables	Set up correct calculations	Organize equations to balance	Calculations	Answer
Student 1	Yes	Adds integer under every symbol	Yes	No, but correct after organization error
Student 2	Yes	Yes	Yes	Yes
Student 3	Yes	Yes	yes	Yes

3. Reteach the student according to what was missed
Student should be taught to organize equations to balance.

Spaced *I*nstructional *R*eview Planning Sheet

Block (Date)	Big Ideas Covered (or specific skills)	Problematic Areas	Problematic Areas Targeted for SIR	Date and Instructional Time Allotted (30-40 minutes)
	<ol style="list-style-type: none"> 1. 2. 3. 4. 5. 	<ol style="list-style-type: none"> 1. 2. 3. 4. 	<ol style="list-style-type: none"> 1. 2. 	
	<ol style="list-style-type: none"> 1. 2. 3. 4. 5. 	<ol style="list-style-type: none"> 1. 2. 3. 4. 	<ol style="list-style-type: none"> 1. 2. 	
	<ol style="list-style-type: none"> 1. 2. 3. 4. 5. 	<ol style="list-style-type: none"> 1. 2. 3. 4. 	<ol style="list-style-type: none"> 1. 2. 	
	<ol style="list-style-type: none"> 1. 2. 3. 4. 5. 	<ol style="list-style-type: none"> 1. 2. 3. 4. 	<ol style="list-style-type: none"> 1. 2. 	

Notes: Use assessment data from Teacher assessments (formal & informal), Progress Monitoring data sources, State Assessments, and other sources of information (teacher's experience). Table abbreviated for space considerations.

Additional Information and Resources

Ashlock, R. B. (2002). *Error patterns in computation: Using error patterns to improve instruction* (8th ed.). Merrill Prentice Hall: OH

Ellis, E. S., Worthington, L., & Larkin, M. J. (1994). *Executive summary of research synthesis on effective teaching principles and the design of quality tools for educators* (Technical Report No. 6, University of Oregon, National Center to Improve the Tools of Educators). Retrieved July 17, 2004, from <http://idea.uoregon.edu/~ncite/documents/techrep/other.html>

Hudson, P., & Miller, S. P. (2006). *Designing and implementing mathematics instruction for students with diverse learning needs*. Boston: Pearson Education, Inc.

Mastropieri, S., & Scruggs, T. E. (2004). *The inclusive classroom: Strategies for effective instruction* (2nd ed.). New Jersey: Pearson Merrill Prentice Hall

Pashler, H., Bain, P., Bottge, B., Graesser, A., Koedinger, K., McDaniel, M., and Metcalfe, J. (2007). *Organizing Instruction and Study to Improve Student Learning* (NCER 2007-2004). Washington, DC: National Center for Education Research, Institute of Education Sciences, U.S. Department of Education. Retrieved from <http://ncer.ed.gov>.

Riccomini, P. J. & Witzel, B. S. (2010). *Response to intervention in mathematics*. Thousands Oaks, CA: Corwin Press.

Riccomini, P. J. & Witzel, B. S. (2010). *Computation of integers: Mathematics interventions for middle and high school students*. Upper Saddle River, NJ: Pearson Education Inc

Stein, M., Kinder, D., Silbert, J., & Carnine, D. W. (2006). *Designing effective mathematics instruction: A direct instruction approach* (4th ed.). Ohio: Pearson-Merrill Prentice Hall.

Witzel, B., & Riccomini, P. J. (2009). *Computation of fractions: Math interventions for elementary and middle grades students*. Upper Saddle River, NJ: Pearson Education Inc.

Witzel, B. S. & Riccomini, P. J. (2011). *Solving algebraic equations: Mathematics interventions for middle and high school students*. Upper Saddle River, NJ: Pearson Education Inc.